$$\max \; 4x_1 + 5x_2$$

s.t.

$$x_1 + 2x_2 \le 8$$
$$x_1 + x_1 \le 5$$
$$2x_1 + x_2 \le 8$$
$$x_1, x_2 \geq 0$$

If $x^* = (2,3)$, $z = 23$

What does complementary slackness say?

Can

$y_1^{(\geq)}$ $1(x_1 + 2x_2 \le 8)$

$y_2^{(\geq)}$ $2(x_1 + x_2 \le 5)$

$y_3^{(\geq)}$ $1(2x_1 + x_2 \le 8)$

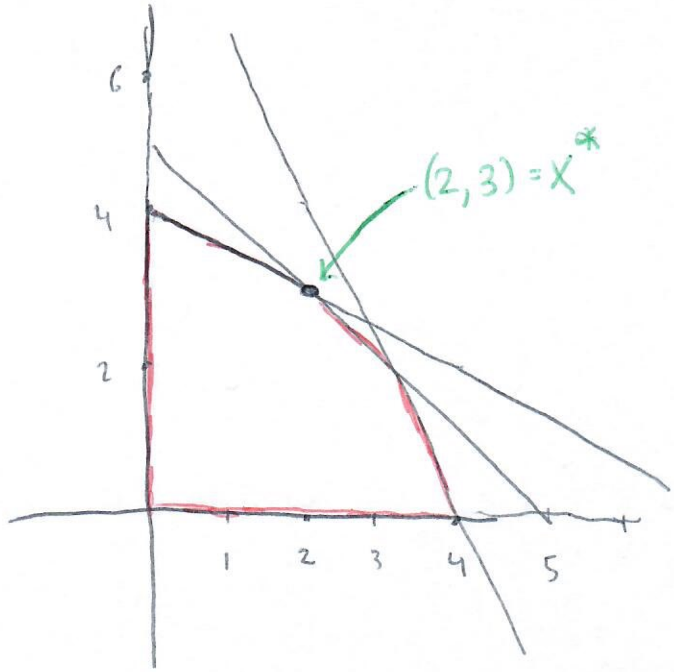
$5x_1 + 5x_2 \le 26$ be optimal upper bound?

(1) Not optimal since we know $x^* = (2,3)$, $z^* = 23$

(2) For $x^* = (2,3)$ says $7 \le 8$ so "complementary slackness" says $2x_1 + x_2 \le 8$ can't be involved in optimal bound

(3) Since $x_1^* > 0$, $4x_1^* + 5x_2^* < 5x_1^* + 5x_2^*$

Other part complementary slackness: if $x_1^* > 0$ the dual constraint $y_1 + y_2 + 2y_3 \geq 4$ must be tight.



$\min \; -4x_1 - 5x_2 = f(x_1, x_2)$

s.t.

$g_1(x_1,x_2) = x_1 + 2x_2 - 8 \le 0$

$g_2: \; x_1 + x_2 - 5 \le 0$

$g_3: \; 2x_1 + x_2 - 8 \le 0$

$g_4: \; -x_1 \le 0$

$g_5: \; -x_2 \le 0$

Complementary slackness vs. Active/Inactive

- $g_3(x^*) = -1$ inactive $\Leftrightarrow$ $2x_1^* + x_2^* = 7 < 8$

  Hence can't involve g_3 in optimal dual upper bound

- $g_4(x^*) = -2$ inactive $\Leftrightarrow$ $x_1^* > 0$

  Hence dual bound must be tight in x_1 dual constraint

- $g_5(x^*) = -3$ inactive $\Leftrightarrow$ similar conclusions.

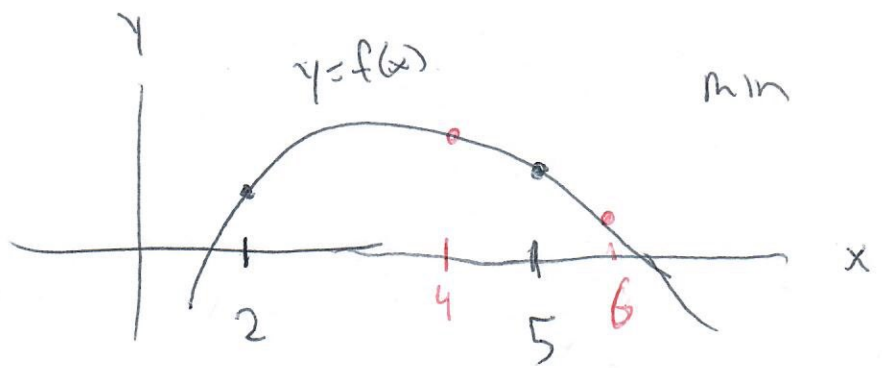
Dual LP shows

$1 \cdot g_1 + 3 \cdot g_2 : \; 3(x_1 + x_2 - 5) + (2x_1 + x_2 - 8) \le 0$

$\Leftrightarrow \; 4x_1 + 5x_2 - 23 \le 0$

$\nabla f = (-4, -5)$ and $1 \nabla g_1 + 3 \nabla g_2 = (4,5)$.

KKT



So you have to check $x = 2, 5$ in min concave down over $2 \le x \le 5$

If $2 \le x_1 \le 5$
$2 \le x_2 \le 5$
$\vdots$
$2 \le x_n \le 5$

have to check 2^n endpoints if $\vec{x} \in [2,5]^n$