Nov 17 (1)

min $f(\vec{x})$ s.t. $g_1(\vec{x}), \ldots, g_m(\vec{x}) \leq 0$

KKT point:

$$(u_0)\nabla f(\vec{x}) + u_1 \nabla g_1(\vec{x}) + \ldots + u_m \nabla g_m(\vec{x}) = 0$$

and $u_1, \ldots, u_m \geq 0$ can be tricky to solve for...

For us simple examples:

~~min $f(w_1, w_2)$ = max~~

Amazon, Bond

$$R = A w_1 + B w_2$$

max $U_{Mark}(\mu; w_1, w_2)$

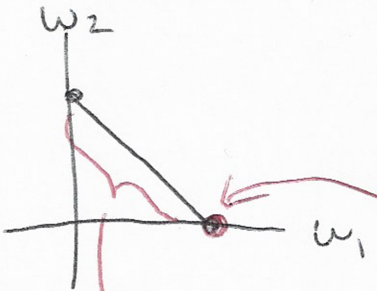
$w_1 + w_2 = 10$

$w_1, w_2 \geq 0$

$$\begin{cases} g_1(w_1,w_2) = w_1 + w_2 - 10 \leq 0 \\ g_2(w_1,w_2) = -w_1 - w_2 + 10 \leq 0 \end{cases}$$

$g_3(w_1,w_2) = -w_1 \leq 0$

$g_4(w_1,w_2) = -w_2 \leq 0$



last time g_1, g_2 active

point $(10,0)$: g_1, g_2, g_4 active

KKT condition at $(10,0)$, i.e. g_1, g_2, g_4 active:

$$(u_0)\nabla f + u_1 \nabla g_1 + u_2 \nabla g_2 + u_4 \nabla g_4$$

(usually $u_0 \neq 0$, $u_0 = 1$)

②

$f(w_1,w_2) = -(10w_1 + 9w_2 - \mu 4 w_1^2)$

$\nabla f = (-10+8\mu w_1, -9)$ $\nabla g_1 = (1,1)$, $\nabla g_2 = (-1,-1)$

systematic $\nabla g_4 = (0,-1)$

KKT: $(-10+8\mu w_1, -9) + u_1(1,1) + u_2(-1,-1) + u_4(0,-1) = 0$

2 equations:

$(-10+8\mu w_1) + u_1 - u_2 + 0 = 0$

$(-9) + u_1 - u_2 - u_4 = 0$

} solve

$u_1 - u_2 = 10 - 8\mu w_1$ $\quad -9+10-8\mu w_1 - u_4 = 0$

$u_4 = -1+8\mu w_1 \geq 0 \Rightarrow 8\mu w_1 \geq 1$

$w_1 = 10 \Rightarrow 80\mu \geq 1, \quad \mu \geq 1/80$

ad hoc