Practical Applications of KKT:

$$\min f(\vec{x}) \quad \text{s.t.} \quad g_1(\vec{x}) \le 0,\ g_2(\vec{x}) \le 0,\ \dots,\ \vec{g_m}(\vec{x}) \le 0$$

$f(\vec{x})$: objective

$g_1(\vec{x}) \le 0,\ g_2(\vec{x}) \le 0,\ \dots,\ \vec{g_m}(\vec{x}) \le 0$: feasibility region

Some Markowitz model applications

$\vec{x}^*$ is a KKT point if

① $\vec{x}^*$ is feasible ② There is a $u = (u_1, \dots, u_m) \ge 0$

s.t. $(\nabla f + u_1 \nabla g_1 + \dots + u_m \nabla g_m)(\vec{x}^*) = 0$

③ If g_i is not active at $\vec{x}^*$, $g_i(\vec{x}^*) < 0$ then $u_i = 0$.

← "Complementary slackness" analogue

Consider $U_m(w_1 X + w_2 Y)$, say $X = A$, $Y = B$

$$U_m(w_1 A + w_2 B) = \overline{w_1 A + w_2 B} - \mu \operatorname{Var}(w_1 A + w_2 B)$$

$\bar{A} = 10$, $\bar{B} = 9$, $\operatorname{Var}(A) = 4$, B no variance ($\operatorname{Var}(w_1 A)$ since B constant)

$$U_m(w_1 A + w_2 B) = w_1 10 + w_2 9 - \mu\, w_1^2 4$$

We imposed: $w_1 \ge 0$, $w_2 \ge 0$, $w_1 + w_2 = 10$

① $\min f = f(w_1, w_2) = -10w_1 - 9w_2 + \mu 4w_1^2$

② Feasibility: $(w_1, w_2) \in \mathbb{R}^2$

$w_1 + w_2 \leq 10$, | $w_1 + w_2 \geq 10$, $w_1 \geq 0$, $w_2 \geq 0$

comes from are equality

$g_1(w_1, w_2) = w_1 + w_2 - 10 \leq 0$

$g_2(w_1, w_2) = -w_1 - w_2 + 10 \leq 0$

$g_3(w_1, w_2) = -w_1 \leq 0$

$g_4(w_1, w_2) = -w_2 \leq 0$

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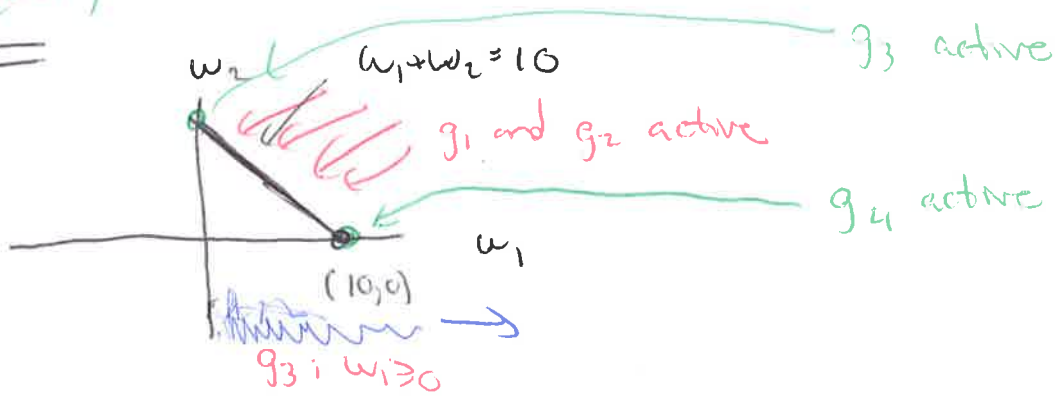

Say that $\vec{x}^*$ is min $f(\vec{x})$ in feasibility region

Say: We are in region where g_1 & g_2 active, and not g_3, g_4

① $g_3(\vec{x}^*), g_4(\vec{x}^*) < 0$

② $(\nabla f + u_1 \nabla g_1 + u_2 \nabla g_2)(\vec{x}^*) = 0$ for some $u_1, u_2 \geq 0$

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$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right)$, $\frac{\partial}{\partial x_1} f$ = differentiate f in x_1, holding x_2 constant

Here $\frac{\partial f}{\partial w_1} = \frac{\partial}{\partial w_1}\left(-10w_1 - 9w_2 + \mu 4 w_1^2\right)$

$= \frac{\partial}{\partial w_1}(-10w_1) + \frac{\partial}{\partial w_1}(-9w_2) + \frac{\partial}{\partial w_1}\left(\mu 4 w_1^2\right)$

$= -10 + 0 + \mu 4 \cdot 2 \cdot w_1$

$$\frac{\partial f}{\partial w_2} = \frac{\partial}{\partial w_2}(-10w_1 - 9w_2 + \mu 4 w_1^2)$$

$$= -9$$

$$\nabla f = \left(\frac{\partial f}{\partial w_1}, \frac{\partial f}{\partial w_2}\right) = (f_{w_1}, f_{w_2}) = (-10 + 8\mu w_1, -9)$$

$g_1 = w_1 + w_2 - 10$ $\quad \nabla g_1 = (1,1)$

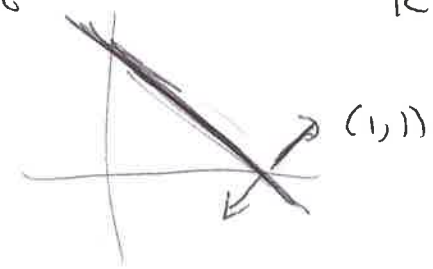
$g_2 = -w_1 - w_2 + 10$ $\quad \nabla g_2 = \left(\frac{\partial g_2}{\partial w_1}, \frac{\partial g_2}{\partial w_2}\right) = (-1,-1)$

KKT: For some $u_1, u_2 \geq 0$:

$$\nabla f \curvearrowright (-10 + 8\mu w_1, -9) + u_1(1,1) + u_2(-1,-1) = 0$$

$$\nabla f = (-10 + 8\mu w_1, -9) = u_2(1,1) - u_1(1,1) = (u_2 - u_1)(1,1)$$

$w_1 + w_2 = 10$



KKT: $\nabla f = \underbrace{(u_2 - u_1)}_{\text{real number}}(1,1)$ (essentially the Lagrange multiplier condition)

So $(-10 + 8\mu w_1, -9)$ proportional to $(1,1)$

So $-10 + 8\mu w_1 = -9$ so $8\mu w_1 = 1$

$$w_1 = 1/(8\mu)$$

So g_1, g_2 must be active, g_3, g_4 not active... $(0 < w_1 < 10)$

So if $0 < \frac{1}{8\mu} < 10$ then $w_1^* = \frac{1}{8\mu}$, $w_2^* = 10 - \frac{1}{8\mu}$ is a KKT point.

3-7 pages (600-1400 words) + appendix + computer software / data attachments (Final Report)

Writing Standards

	Excellent	Good	Satisfactory	Poor
Abstract and Introduction Weight = 1	• Excellent overview of the background is given • Motivation for studying the problem is credible and convincing • Purpose of the paper is clearly explained and justified	• Some overview of the background is given • Motivation is valid, but incomplete and possibly not effective • Purpose of the paper is stated	• Background is incomplete • Motivation is vague and not supported • Purpose of the paper is obscure	• No background given • Motivation is unclear • Purpose of the paper is vague or not explicitly stated
Overall organization Weight = 1	• Logically organized • Sections and results are clearly marked • Plenty of appropriate links and transitions	• Paper is mostly well organized • Most transitions are present	• Some sections are too long/short or are not logically placed • Incorrect or missing transitions	• Section and result names and/or numbers are missing • No transitions and no overall flow
Writing style Weight = 1	• Tone is professional • Audience is addressed appropriately with uniform amount of detail	• Tone is appropriate • Most details are appropriate for the intended audience	• Tone is mostly respectful • Some details are skipped or redundant	• Tone is patronizing, disrespectful • Details are lacking or redundant
Modeling terminology and content Weight = 5	• All necessary variables, constraints, and objectives are given and explained • All optimization problems correspond to the goals of the project • Sources of data and any synthetic generation of data are explained and realistic • Optimization algorithms and methods of analyzing the results are explained. • Explanations are correct, clear, and easy to follow	• Some notation and explanations are not given • Most of the optimization problems are correctly formulated and consistent with the goals of the project • The stated results in the article are mostly justified by the optimization problems solved. • Most explanations are precise and can be followed	• A number of explanations are not given • Some terminology is not correctly used • The optimization problems studied and the data correspond only weakly to the results claimed in the article • Reasoning and explanation are lacking	• Notation is not defined • Terminology is misused • Modeling is incorrect • Results claimed aren't justified by the data and optimization problems solved
Spelling and grammar Weight = 1	• Excellent grammar, perfect spelling • Strong sentences • Math is well incorporated into the text	• Mostly good grammar, perfect spelling • Math is mostly well integrated into the text	• Few awkward sentences • Some spelling mistakes • Math is separated from the text	• Awkward phrases • Many spelling mistakes • Math is not integrated or punctuated
Bibliography and citations Weight = 1	• Excellent use of citations • Bibliography is complete with uniformly formatted entries	• Most citations are present • Some bibliography items are incorrectly formatted	• Citations are missing • Bibliography has errors, and some necessary sources are not listed	• Few or no citations • Bibliography entries are not clear or missing

Reminders for Final Projects:

- Deadline: email by Tuesday, Nov 28, 11:59 pm
  written report " Wednesday, " 29, in class.

  [Faculty of Science regulation: Projects cannot be accepted after last class.]

- Bibliography will be graded;
- Make sure model equations are correct, without even minor errors.