Oct 8

- Talk about KKT conditions, convex prog.
- Section 6.3 of Markowitz model; basis for Homework tomorrow

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$R = w_1 R_1 + w_2 R_2 + \dots + w_n R_n$ n financial instruments

$n=2$; $R = w_1 X + w_2 Y$, condition $W_1 + W_2$ is given

In §6.3: (1) $R = w_1 A + w_2 M$ (Money, M, $\overline{M}=0$, $Var(M)=0$)

(2) $R = w_1 A + w_2 B$ (B = Bond, $\overline{B}=9$, $Var(0)$)

$\{ w_1 + w_2 = 10$, $w_2 = 10 - w_1$ substitute $0 \le w_1 \le 10$

In general: $U_{mark}(\mu, R) = \overline{R} - \mu \, Var(R)$; $R = w_1 X + w_2 Y$

$$U(\mu; w_1, w_2) = \overline{\cancel{X} w_1 + Y w_2} - \mu \, Var(X w_1 + Y w_2)$$

$$= w_1 \overline{X} + w_2 \overline{Y} - \mu \left(w_1^2 Var(X) + w_2^2 Var(Y) + 2 w_1 w_2 \, Cov(X, Y) \right)$$

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(1) $R = w_1 A + w_2 M$

$$U = w_1 \overline{A} + w_2 \overline{M} - \mu \left(w_1^2 Var(A) + w_2^2 Var(M) + 2 w_1 w_2 \, Cov(A, M) \right)$$

Recall Model: $\overline{A} = \overline{N} = \overline{C} = \overline{P} = 10$, $\overline{B} = 9$, $\overline{M} = 0$

$Var(M) = Var(B) = 0$, $Var(A) = Var(N) = Var(C) = Var(P) = 4$

If Y has $Var(Y)=0$, then

$$Var(w_1 X + w_2 Y) = Var(w_1 X) = w_1^2 \, Var(X)$$

If $Var(X) \neq 0$, $Var(Y) \neq 0$

$$Cov(X,Y) \left(= \overline{XY} - \bar{X}\bar{Y}\right) = \underbrace{Corr(X,Y)}_{\text{between } -1 \text{ and } 1} \; \underbrace{Var(X)\, Var(Y)}_{(=16 \text{ if } X,Y = A,N,C,P)}$$

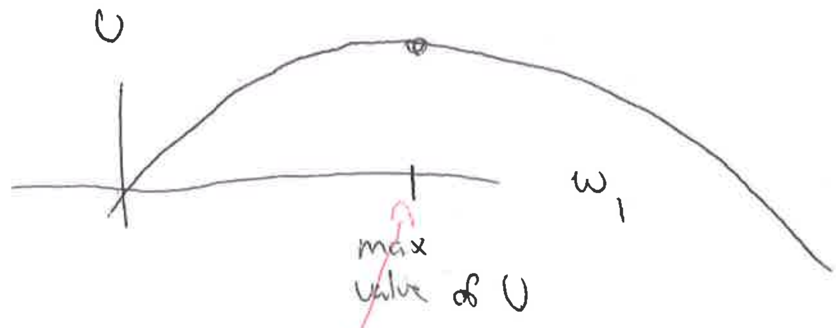
(1) $R = w_1 A + w_2 M$:

$$U_{mark}(\mu;\, w_1 A + w_2 M) = w_1 \bar{A} + w_2 \bar{M} - \mu \, Var(w_1 A + w_2 M)$$

$$= w_1 10 + w_2 0 - \mu \, w_1^2 4 \qquad (4 = Var(A))$$

$$U = 10 w_1 - \mu w_1^2 4$$

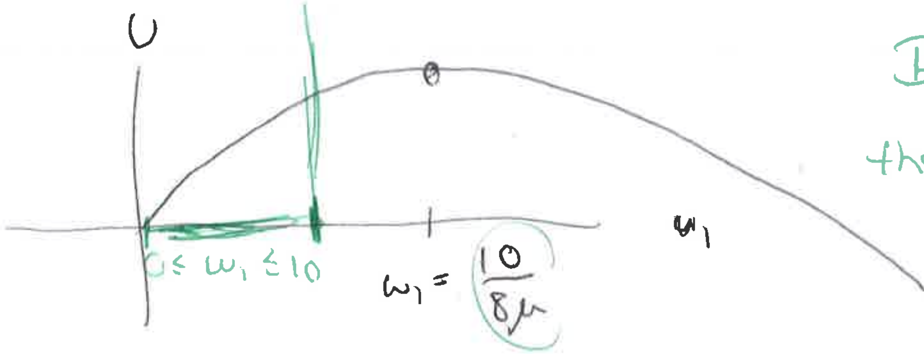
$(0 \leq w_1 \leq 10)$



$$\frac{dU}{dw_1} = 10 - 8\mu w_1$$

is zero when $8\mu w_1 = 10$

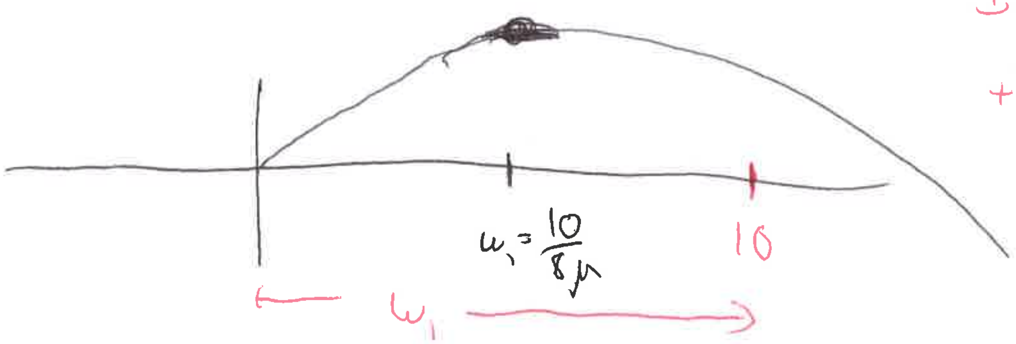
$$w_1 = 10/8\mu$$



If $10 \le \frac{10}{8\mu}$

then

$w_1^* = 10$



If $\frac{10}{8\mu} \le 10$

then

$w_1^* = \frac{10}{8\mu}$

So, more succinctly, $w_1^* = \text{Min}\left(10, \frac{10}{8\mu}\right)$

(2) in Section 6.3:

$R = w_1 A + w_2 B$;

$$U = w_1 \bar{A} + w_2 \bar{B} - \mu\left(\text{Var}(w_1 A + w_2 B)\right)$$

$$= w_1 10 + w_2 9 - \mu w_1^2 \text{Var}(A)$$

$$= w_1 10 + w_2 9 - \mu w_1^2 4 \qquad (w_2 = 10 - w_1)$$

$$= 10 w_1 + 9(10 - w_1) - \mu w_1^2 4$$

$$= 90 + w_1 - \mu w_1^2 4 \quad \leftarrow \text{for } w_1 A + w_2 B$$

vs $\quad 10 w_1 - \mu w_1^2 4$ for $w_1 A + w_2 M$