Nov 1

- Article Markowitz Model ✓
- 1998 $\underbrace{\text{Linear Complementarity}}_{\text{Quadratic Programming (+)}}$ and $\underbrace{\text{Mathematical Prog}}_{\text{Any old optimization}}$

Sections 1–5: Quadratic Programming, LP

Section 6: KKT Conditions (local conditions)

7: More " "

8: In convex optimization KKT is $\left\{\begin{array}{l}\text{necessary}\\ \& \\ \text{sufficient}\end{array}\right.$

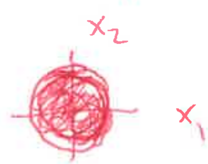
⤳ algorithms

General Optimization:

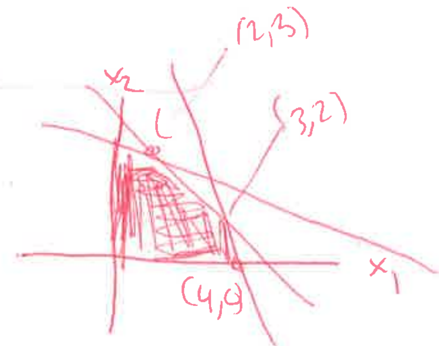
$$\underline{\min}\ \underset{\text{objective}}{f(x)},\quad \underbrace{g_1(\vec{x}) \le 0,\ g_2(\vec{x}) \le 0, \ldots, g_m(\vec{x}) \le 0}_{\text{constraints}}$$

"Convex optimization" ⟷ convex regions

e.g. $\{\ x_1^2 + x_2^2 \le 1 \iff x_1^2 + x_2^2 - 1 \le 0$



e.g. $x_1 + 2x_2 \le 8$, $x_1 + x_2 \le 5$, $2x_1 + x_2 \le 8$, $x_1, x_2 \ge 0$



KKT Conditions: We say $\vec{x}_0$ is a KKT point if

- LP duality
- Lagrange mult.
- Unconstrained

$g_1(\vec{x}_0), \ldots, g_m(\vec{x}_0) \le 0$, $(\nabla f + u_1 \nabla g_1 + \ldots + u_m \nabla g_m)(\vec{x}_0) = 0$

the $u_1, \ldots, u_m \ge 0$ reals (dual variables in LP), and if $g_i(\vec{x}_0) < 0$ then $u_i = 0$.

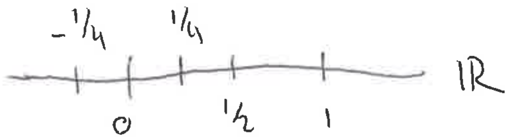
§8: If $f, g_1, \ldots, g_m$ are convex then KKT conditions are $\left\{\begin{array}{l}\text{necessary}\\ \text{sufficient}\end{array}\right.$ for $\vec{x}_0$ optimal feasible solution.

Examples:

One dimension = ...

Everything will reduce to one dimension...

Convexity:



$\mathbb{R} \to \mathbb{R}^2 \to \dots$

$\to \mathbb{R}^2$

$\frac{5}{4}\vec{a} + \left(\frac{-1}{4}\right)\vec{b}$

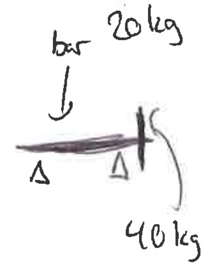
$(125\%)\vec{a} + (-25\%)\vec{b}$

$(75\%)\vec{a} + (25\%)\vec{b}$

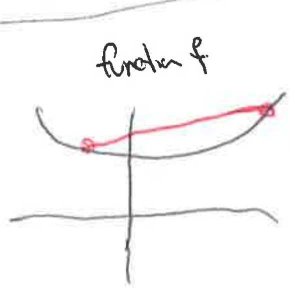
$= \frac{3}{4}\vec{a} + \frac{1}{4}\vec{b}$

$\frac{\vec{a}+\vec{b}}{2} = (50\%)\vec{a} + (50\%)\vec{b}$

$\to$ centre of mass



Convex:



$(\vec{x}, f(\vec{x}))$

line segment above function value

$\vec{x} \in \mathbb{R}^n$