- Finish Var/Covar
- Start KKT points / Quadratic Optimization

Start article in § 5-8

(Rem: Examples 5.2, 5.3 in § 5 are Markowitz model)

Example 5.2: $\omega \in \mathbb{R}^n$, minimize $Var(R(\vec{w}))$ s.t. conditions ... and $\overline{R(\vec{w})} = r_0$ given

$U_m = \overline{R(\vec{w})} - \mu \, Var(R(\vec{w}))$ (maximize)

Literature: minimize $f(\vec{x})$ s.t. $\vec{g}(\vec{x}) \leq 0$.

Last time: C, P, A, N : all had same cost, same expected value 10, same variance 4

B : $\overline{B} = 9$, $Var(B) = 0$.

$Cov(X,Y) = \overline{XY} - \overline{X}\,\overline{Y}$

– $Var(X) = \overline{X^2} - (\overline{X})^2 = \overline{(X-\overline{X})^2} = Cov(X,X)$

Intuition:

$$Correlation(X,Y) = Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}}$$

assume non-zero

Fact:

$$-1 \le Corr(X,Y) \le 1$$

1. $Corr(X,X) = 1$: $\dfrac{Cov(X,X)}{\sqrt{Var(X)\,Var(X)}} = \dfrac{Var(X)}{Var(X)} = 1$

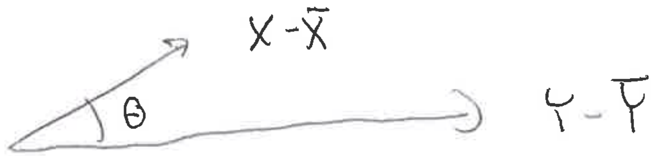
2. $Corr(X,-X) = -1$

3. $Corr(X,2X) = Corr(5X,7X) = Corr(-3X,-12X) = 1$

   $Corr(X,-2X) = Corr(-5X,7X) = Corr(-3X,12X) = -1$

4. $Corr(X,Y)$:

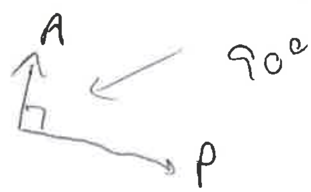
   $Corr(X,Y) = \cos(\theta)$



$C = P$: $Corr(C,C) = Corr(C,P) = Corr(P,P) = 1$

Amazon: Independent of P

$$\boxed{Corr(A,P) = 0}$$



$Var(A+P) = Var(A) + Var(P) + 2\,\cancel{Cov(A,P)} = 2\,Var(A)$

$Var(A+A) = Var(A) + Var(A) + 2\,Cov(A,A) = 4\,Var(A)$

$Var(A+N) = Var(A) + Var(N) + 2\,Cov(A,N)$

$= 4 + 4 + 2(-4) = 0$

$Var(A+(-A)) = 4 + 4 + 2(-4) = 0$

maximize $U_m(R(\vec{w}))$

$$= \overline{R(\vec{w})} - \mu \, Var(R(\vec{w}))$$

$$= \sum_{i=1}^{n} w_i \overline{R_i} - \mu \sum_{i,j=1}^{n} Cov(R_i, R_j)\, w_i w_j$$

= so what?

[Constraints: Linear: budget: $\sum w_i \, Cost_i = Budget$]

Now we look at

minimize $f(\vec{x}) = f(x_1, \dots, x_n)$

s.t. $\vec{g}(\vec{x}) \le \vec{0}$: $g_1(x_1, \dots, x_n) \le 0$

$\vdots$

$g_m(x_1, \dots, x_n) \le 0$

We want to extend theory / algorithms for LP to ... quadratic programming ... we'll need conditions on $f, g_1, \dots$

LP: max $\vec{c}^T \vec{x}$, minimize $f(\vec{x}) = -\vec{c}^T \vec{x}$

s.t. $A\vec{x} \le b$, $\vec{x} \ge 0$ $\Leftrightarrow$ $A\vec{x} - b \le 0$

$-\vec{x} \le 0$

Oct 30

There's a way to solve LP without LP...

Complementary slackness...

max $4x_1 + 5x_2$ s.t.

Dictionaries duality...

$$x_1 + 2x_2 \le 8 \quad \leftarrow \cdot 1$$
$$x_1 + x_2 \le 5 \quad \leftarrow \cdot 3$$
$$2x_1 + x_2 \le 8 \quad \leftarrow \cdot 0$$
$$x_1, x_2 \ge 0$$

$4x_1 + 5x_2 \le 23$

optimal ?

Complementary slackness: In LP sufficient & necessary for optimality