- Progress reports due Tuesday 11:59 pm via email, + hardcopy in class on Wednesday
- Monday morning is last possible office hours
- Progress reports 3-5 pages / 600-1000 words

Look at Markowitz model:
- Understand when it's useful / and its limitations...

[Use the fictitious C, P, A, N, B (and other)].

Portfolio $R(\vec{w})$: random variables $R_1, \ldots, R_n$, such as $\underbrace{C, P, A, N}_{\text{risky stocks}}, \underbrace{B}_{\text{bond}}$ (financial instruments)

$$R(\vec{w}) = w_1 R_1 + w_2 R_2 + \ldots + w_n R_n$$

w_i = variables,
R_i = given, we know thing about them

$$U_{Markowitz}(R(\vec{w}), \mu) = \overline{R(\vec{w})} - \mu \, Var(R(\vec{w}))$$

① μ measure of risk aversion

② $$U_m(R(\vec{w})) = \sum_{i=1}^{n} w_i \overline{R_i} - \mu \sum_{i,j=1}^{n} w_i w_j \, Cov(R_i, R_j)$$

③ limitations & ~~positive aspects~~ advantages of Markowitz model

Say A = Amazon, Bond = B cost the same amount

$$U_m(A,\mu) = \overline{A} - \mu \operatorname{Var}(A) = 10 - \mu 4$$

$$U_m(B,\mu) = \overline{B} - \mu \operatorname{Var}(B) = 9 \qquad (\operatorname{Var}(B)=0)$$

So

$$10 - 4\mu \text{ vs } 9: \quad \begin{cases} \mu>0,\ \mu<1/4, \text{ prefer } A \\ \mu>0,\ \mu>1/4 \text{ prefer } B \end{cases}$$

Indicates: large positive $\mu \Rightarrow$ averse to "risk"
small positive $\mu \Rightarrow$ tolerant of risk

$\mu < 0$ in Casinos! G = Game, if $\overline{G} < 0$, $\operatorname{Var}(G)$ big ☺

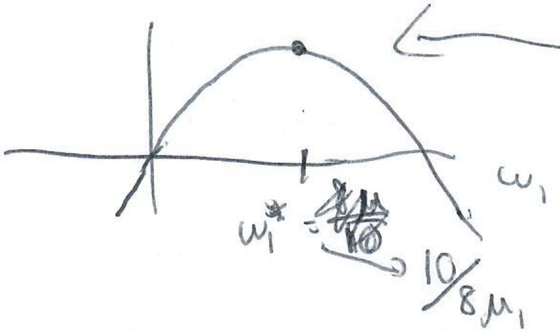
Now let's ... situation: $R(\vec{w}) = w_1 A$, $n=1$ financial instruments.

Sensible?

$$U_m(R(\vec{w})) = U_m(w_1 A) = \overline{A w_1} - \mu \operatorname{Var}(A w_1)$$

$$= \overline{A} w_1 - \mu \operatorname{Var}(A) w_1^2$$

$$= 10 w_1 - \mu 4 w_1^2$$



$(\mu>0)$

$$\frac{d}{dw_1}(10w_1 - \mu 4 w_1^2) = 0$$

$$10 = 4\mu 2 w_1$$

$$w_1 = \cancel{8\mu/10}\ \frac{10}{8\mu_1}$$

Markowitz model requires:

- 2 or more instruments, (one which is pretty stable) (and tradie trades off)
- A fixed budget, and

$$\boxed{\sum w_i \, (\text{Cost } R_i) = \text{budget}}$$

- Or you have an instrument:

M = store money under your mattress

$$\boxed{R(\underbrace{w_1 A + w_2 N + w_3 B}\, + w_4 \underbrace{M}_{\text{money}})}$$

if $w_1 \text{Cost}_1 + w_2 \text{Cost}_2 + w_3 \text{Cost}_3 \leq \text{budget}$

$w_4 = \quad w_1 \quad w_2 \quad w_3$

Test: ① $R = w_1 A + w_2 B$, $\quad$ A, B same cost, so

$$\boxed{w_1 + w_2 = \text{given} = (\text{your budget}/\text{Cost})}$$

② $R = w_1 A + w_2 N \quad \leftarrow N = \text{fiction}$.

take $w_1 + w_2 = 30$

$\overline{R} = w_1 \overline{A} + w_2 \overline{N} = w_1 10 + w_2 10 = 10(w_1 + w_2) = 10 \cdot 30 = 300$

In general, assume A, N, C, P, B cost the same

$$U(w_1 A + w_2 N) = \overline{w_1 A + w_2 N} - \mu \, \text{Var}(w_1 A + w_2 N)$$

$$= 300 - \mu \underbrace{Var(w_1 A + w_2 N)}_{= ???}$$

We define, random variables X, Y, $Cov(X,Y) = \overline{XY} - \bar{X}\,\bar{Y}$

Formulas:

① $Var\,X = \overline{(X-\bar{X})^2} = \overline{(X^2 - 2X\bar{X} - \bar{X}^2)}$

$$= \overline{X^2} - 2\,\overline{(X\bar{X})} + \overline{(\bar{X})^2}$$

$$= \overline{X^2} - 2\,\bar{X}\,\bar{X} + (\bar{X})^2 = \overline{X^2} - (\bar{X})^2$$

② $Cov(X,X) = Var(X)$ (!)

③ Real motivation:

$$Var(X+Y) = \overline{\left(X+Y - \overline{(X+Y)}\right)^2}$$

$$= \overline{X^2+Y^2+2XY - \text{other stuff}} = ..$$

$$= \overline{X^2} - (\bar{X})^2 + \overline{Y^2} - (\bar{Y})^2 + 2\left(\overline{XY} - \bar{X}\,\bar{Y}\right)$$

$$= Var\,X + Var\,Y + 2\,Cov(X,Y)$$

Fact: $Var(A) = Var(N) = 4$ but $Cov(A,N) = -4$

$Var(A+N) = \ldots = 0$