Oct 23 ①

Markowitz Model, Non-linear (quadratic programming)

In preparation (Ch 24 Vanderbei) — Posted article

Quadratic Programming:

maximize/minimize: $U = U(\vec{x}) = \underbrace{\vec{c}^T \vec{x}}_{\text{linear}} + \underbrace{\sum_{i,j} a_{ij} x_i x_j}_{\text{deg } 2}$

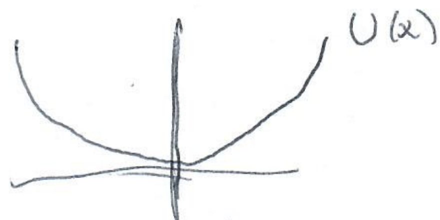
s.t. $A\vec{x} \le b$ ⟵

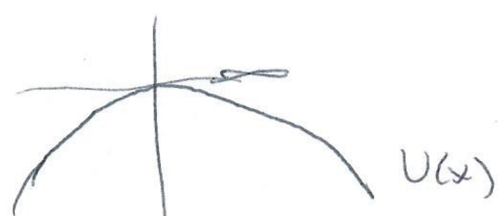
maybe $\vec{x} \ge 0$ ⟵ Focus on

[maybe $\vec{x} \in \mathbb{Z}^n$ (integers)]

Look at max/min $\vec{x} = (x)$ one variable

maximize $U = x^2$ — maximize $U = -x^2$





unbounded ⟵ x free, $x \in \mathbb{R}$ ⟶ bounded

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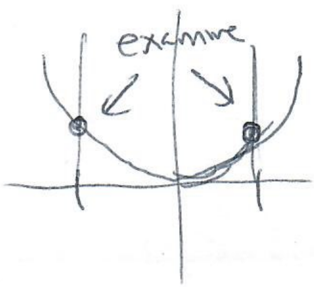
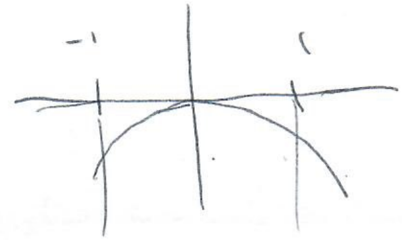
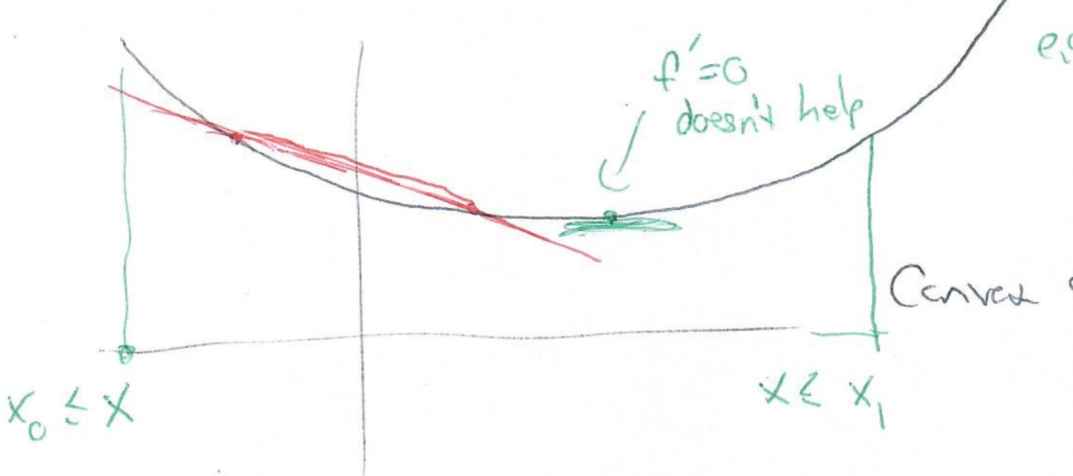
say $-1 \le x \le 1$



(convex)



Maximize convex function:



e.g. $f(x) = x^2$

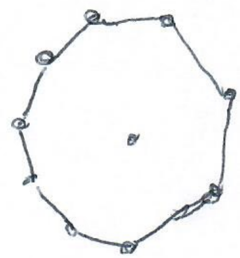
Convex $\Leftrightarrow$ $f'' > 0$

Worse with 2 variables:

$f(\vec{x}) = f(x_1, x_2) = x_1^2 + x_2^2 = (\text{distance to } (0,0))^2$

say:

perturb a little



$\leftarrow$ $A\vec{x} \le b$ (forget $\vec{x} \ge 0$)

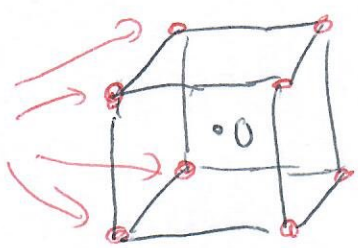
have to examine each corner

Look at $-1 \le x_1 \le 1$, $-1 \le x_2 \le 1$, ..., $-1 \le x_n \le 1$

Look at $f(\vec{x}) = x_1^2 + x_2^2 + \dots + x_n^2$

2n inequalities

$f = 3$

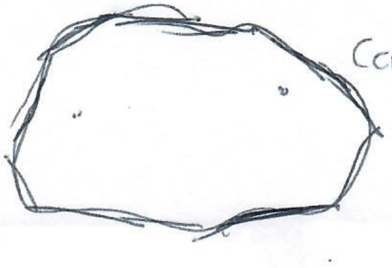


2^n vertices

locally must check everything

Claim: If $f(\vec{x})$ is concave down,

$\vec{x} \in \mathbb{R}^n$



convex,

local properties can guarantee global maximum