October 18 (1)

- Finish scheduling with wait times
- Start Markowitz model, quad programming
- Remarks on homework (due by Thursday)

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HW 2: LP: x_1, x_2: $\max \vec{c}^T \vec{x}$ s.t. $A\vec{x} \le b$, $\vec{x} \ge 0$, $\vec{x} \in \mathbb{Z}$

(See Sept 20 examples)

HOMEWORK #2, MATH 441, FALL 2017

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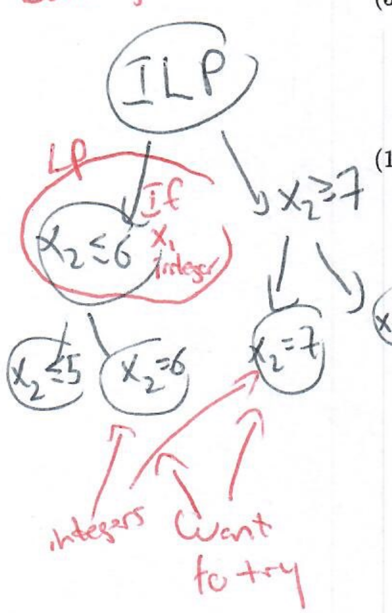
Please note:

(1) You may work together on homework, but you must write up your own solutions individually. In particular, you must write your own code, spreadsheets, etc.
(2) You must acknowledge with whom you worked (specify their `gradescope.com` email addresses). You must also acknowledge any sources you have used beyond the textbook and class material.
(3) When you submit your homework to `gradescope.com`, you need to put the solutions to different problems on different pages; `gradescope.com` will ask you to identify which pages correspond to which problems.

Bounding: via LP



(1) Use branch and bound to solve the integer linear program max $\vec{c}^T\vec{x}$ subject to $A\vec{x} \le \vec{b}$ and $\vec{x} \ge 0$ and $\vec{x} \in \mathbb{Z}^2$ (i.e., x_1, x_2 must be integers)

$$\vec{c} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 29.8 \\ 7.3 \\ 8.2 \end{bmatrix}.$$

Do not make use of the specific properties of $A, \vec{b}, \vec{c}$ in this problem (i.e., that they all have non-negative entries/coefficients). Specifically:

(a) Enter the corresponding LP into your LP software; you should find that the optimum solution is $x_1 = 0.9$, $x_2 = 6.4$, and $z = 22.8$.
(b) Try the following branches: $x_2 \le 6$ and $x_2 \ge 7$. If you need to explore the $x_2 \le 6$ branch further, divide this branch into $x_2 \le 5$ and $x_2 = 6$; if you need to explore the $x_2 \ge 7$ branch further, divide this branch into $x_2 = 7$ and $x_2 \ge 8$. (You should find that the branch $x_2 \ge 8$ is infeasible.)
(c) When you reach a branch with x_2 fixed, branch on x_1 in a similar fashion (solve the relaxed LP, and round up and down).
(d) Complete the branch and bound, and make a diagram of the result.

LP
$x_2 = 6$
$x_1 = 3.2$

maybe
$x_2 = 5$
$x_1 = 4$

better

$x_2 = 6$
$x_1 = 3$

Research supported in part by an NSERC grant.

– Soon start Markowitz model, quad prog.

(Refs: Vanderbei, Ch 24; old handout of mine)

Before that: scheduling:

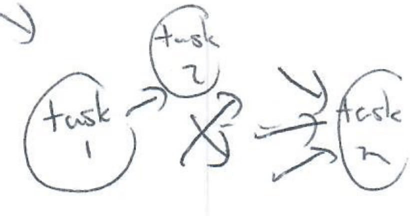
added

min $x_n - x_1$ s.t.

$x_i + Wait(i,j) \le x_j$ | $(i,j) \in E$

x_i's are real, unbounded

visualize



Wait times might be: 1, 20, 3, 3.721, 5.98317, ... not necessarily integers

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Dual:

$$\sum_{i,j} y_{ij} \left(x_i + Wait(i,j) \le x_j \right)$$

$$\sum y_{ij} W(i,j) \le \sum y_{ij}(x_j - x_i) = x_n - x_1$$

benefit — for dual LP

So dual:

$\sum_{i,j} y_{ij} W(i,j)$ is a lower bound on $x_n - x_1$

If $\sum y_{ij}(x_j - x_i)$ gives $x_n - x_1$

so take x_k coef:

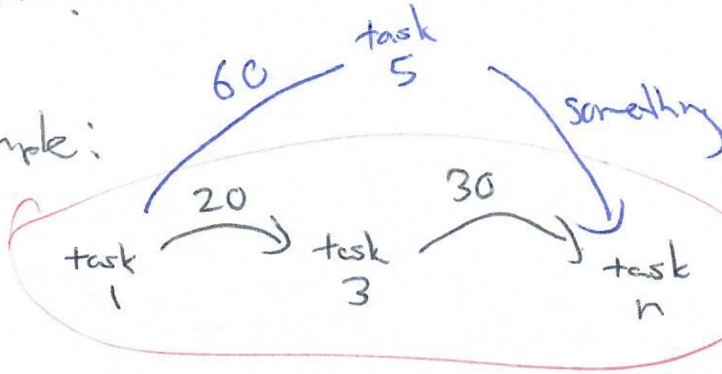
for x_k, task k

Dual constraints

$$\boxed{\sum_i y_{ik} - \sum_i y_{ki} = \begin{cases} 1 & \text{if } k = n \\ -1 & \text{if } k = 1 \\ 0 & \text{if } k = 2, \ldots, n-1 \end{cases}}$$

Done!

Example:



choice

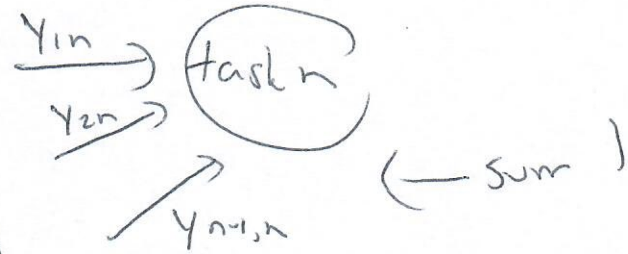
$$Y_{13} = 1 \cdot (x_1 + 20 \le x_3)$$

$$Y_{3n} = 1 \cdot (x_3 + 30 \le x_n)$$

$$x_1 + x_3 + 50 \le x_3 + x_n$$

$$x_1 + 50 \le x_n$$

$$50 \le x_n - x_1$$



must have integer solution by Theorem (total unimod)

$Y_{ij} \ge 0$ so $Y_{1n} + \dots + Y_{n-1,n} = 1$

≥ 0 integers