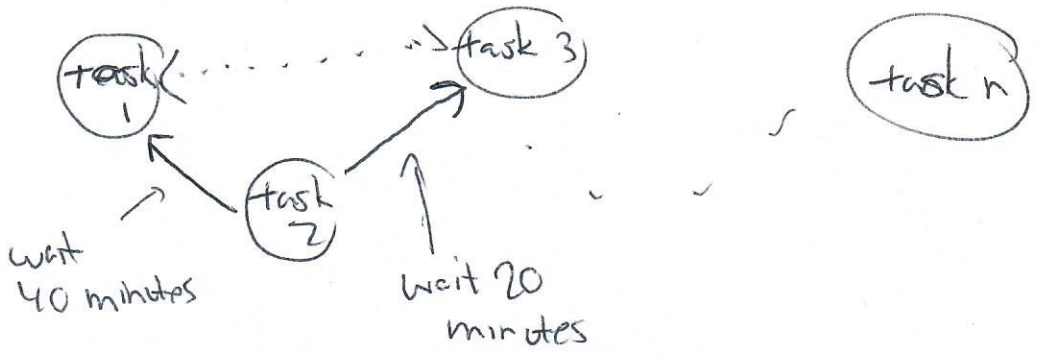
- Proposals coming back today
  - Scores: 10/10, RESUBMIT } Start your project. If RESUBMIT please RESUBMIT.
  - All topics were fine. Start making your prototypes. Start simple...
  - Please submit a hardcopy
- One standard problem:
  - questions sometimes limited to one model, rather than trying out ~~a few~~ models
  - try to ask for what principles your modelling suggests (problem with a lot of data)

you will learn as you start running tests

  - basic model: LP, ILP
- Human experiments: surveys, etc.
  - may be difficult to obtain (unless available from some other source)
  - try to do something where you don't have to interview/surveys/etc.

- Scheduling with time restrictions
- Simple example of where dual is important.



So have graph! Vertices $\{1, \dots, n\}$

Edges are directed: $i \longrightarrow j$

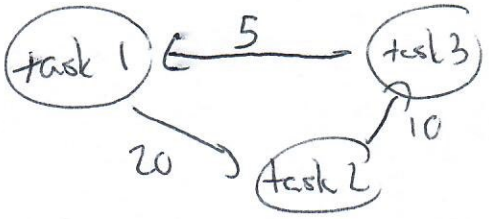
" come with "wait times"

Last time: minimize (max time that a task is performed)

x_i = time when task i is scheduled

Constraint: $x_i + \text{WaitTime}(i,j) \leq x_j$ for $(i,j) \in E$

given given

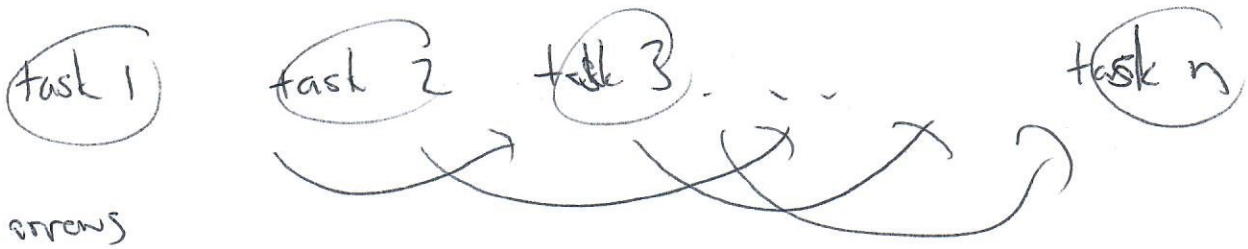


infeasible

"directed cycle"

- This LP, is infeasible if this happens
- If there are no cycles, then: - some task has no arrows out of it
  - " " " " " into it

If we want, by repeating:



all arrows

So constant: $x_i + \text{WaitTime}(i,j) \leq x_j \quad (i,j) \in E$, $i < j$

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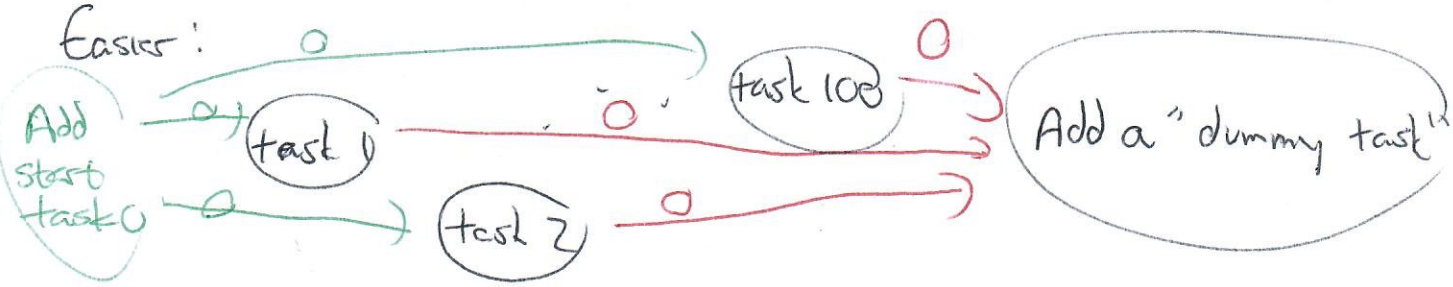
minimize $\max(x_1, \ldots, x_n)$ ← true, but messy...

Easier:



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Assuming such a situation:

Reals $x_1, \ldots, x_n$, ~~min~~ x_n s.t.

$x_i + w(i,j) \leq x_j \quad (i,j) \in$

=

Really want: $\min(x_1, \ldots, x_n)$ vs. $\max(x_1, \ldots, x_n)$

Really want $\min(x_n - x_1)$ assuming x_n = last dummy task
x_1 = first " "

This make dual LP simple!

Original LP:

$$\min (x_n - x_1)$$

s.t.

$$x_i + \text{Wait}(i,j) \le x_j$$

(no bounds $x_1, \ldots, x_n \ge 0$) x_i {free / unbounded}

Turns out! standard form:

$Ax \le b$ ← $y^T A x \le y^T b$ get bound

$x \ge 0$

bound $c^T x$ above

$y^T A \ge c^T$ ← here $x \ge 0$ so is OK

So if $Ax = b$: y unbounded

$x \ge 0$ goes away → $y^T A = c^T$ ← $4x_1 + 5x_2 \le 5x_1 + 5x_2$ if $x_1 \ge 0$ but not if x_1 unbounded

Vanderbei, Chvatal

So dual of

$\min x_n - x_1$

s.t.

$x_i + \text{Wait}(i,j) \le x_j \quad (i,j) \in E$ mult by Y_{ij} for $(i,j) \in E$

$$\sum_{(i,j) \in E} Y_{ij} (\text{Wait}(i,j)) \le (x_j - x_i) Y_{ij}$$