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- ~~H~~omework 2 ~~will~~ will be assigned ~~before~~ after ~~Thanksgiving~~

- Tuesday 11:59 pm
(due date for projects)

- Proposal! - Basic idea (fixed)
  - Basic LP, ILP,.. (fixed)
  - Fundamental Questions/Motivation (can vary)
  - List of 3 or more questions/directions (can vary a lot over time)

e.g. - Exam scheduling (Basic Idea) ← fixed
- Basic LP, ILP; graph colouring ← fixed, will be refined
- ~~F~~undamental Questions/Motivation! Given resources for exam (instructors time), students/faculty/TA happier with {shorter / longer} exam period, given policies that university can set, ~~how does~~ which exam period lengths & conflicts can be achieved.
- List of questions - ⟷ principles
  - what happens if large classes have individual section exams
  - say we want long gaps for students between exam times

might vary over time

Practical Examples:

middle — LP resources products where # each products has to be integer

$\rightsquigarrow$ ILP, probably branch and bound, bounding is based on the LP relaxation!

one extreme — ILP's that can be solved directly from the LP relaxation:

Weighted Bipartite Matching:

max $\vec{c}^T \vec{x}$ s.t.

$A\vec{x} \le \vec{b}$, $\vec{x} \ge 0$

~~($\vec{x}$ components are integers)~~

Claim: If we solve — — — without integral constraints LP, the simplex method always find $\vec{x}$ optimal and integral

the other extreme — Also: discuss bin packing, where take ILP $\rightsquigarrow$ LP relaxation does nothing

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Why do some problems:

max $\vec{c}^T\vec{x}$

s.t. $A\vec{x} \le \vec{b}$

$\vec{x} \ge 0$ with $\cancel{\vec{x} \in \mathbb{Z}^n}$

can you just solve relaxed LP?

The simplex method $\rightsquigarrow$ dictionaries

$$[A \mid I]\begin{bmatrix} \vec{x}_{dec} \\ \vec{x}_{slack} \end{bmatrix} = \vec{b}$$

and

A_B, A_N some cols of $[A_{orig} \mid I]$

$$A_{Basic}\vec{x}_B + A_N\vec{x}_N = \vec{b}$$

$$\boxed{\vec{x}_B = A_B^{-1}(\vec{b} - A_N\vec{x}_N)}$$

Theorem: If $\vec{b}$ has integer components, and A_B is any set of columns of $[A \mid I]$ that is square and A_B has an inverse, then $A_B^{-1}\vec{b}$ will have integer components

if

(every minor of $A = A_{orig}$ has determinants $0, 1, -1$.
"totally unimodular"

max $4x_1 + 5x_2$

$x_1 + 2x_2 \le 8$

$x_1 + x_2 \le 5$

$2x_1 + x_2 \le 8$

$$\left[\begin{array}{cc|ccc} 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix}$$

say basic x_1, x_2, x_3

$$A_B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

from A ... from I

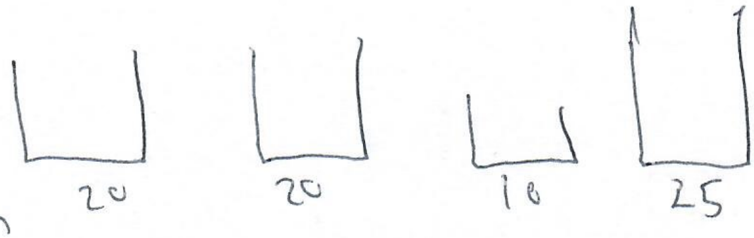
so $\det A_B = 0, 1, -1 \Leftrightarrow \det \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = 0, 1, -1$

Upshot: Look if some book/paper says your A is "totally unimodular." E.g. bipartite matching, network flows, etc.

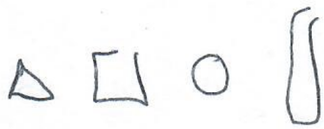
Minors of $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$: 2×2 minors: pick 2 rows, 2 cols:

Bad news for bin packing:

We have some bins

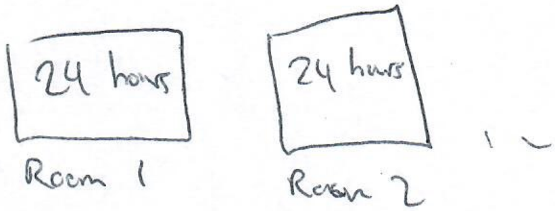


each bin has capacity

Have objects to put in bins  ← each with size

Want to place each item in a bin.

Hospital Operating Rooms



Some operation to perform: 3 hours, 5 hours, 2 hours, ... etc.

Say we have 2 rooms, equal number of hours

Room 1: Capacity 1,
Room 2: Capacity 1,

object to place rooms $a_1, a_2, \ldots, a_n \in [0,1]$

Choose $x_1, \ldots, x_n$ ~~(Assume~~

$x_1 a_1 + \cdots + x_n a_n = 1 - \text{spare}$

min spare

$x_1, \ldots, x_n, \text{spare} \geq 0,$

$x_1 \leq 1, x_2 \leq 1, \ldots, x_n \leq 1$

What happens if

$\vec{x} \in \mathbb{Z}^n$