- Class now!
  - New topics (maybe your project's topic...)
  - Project Related Stuff

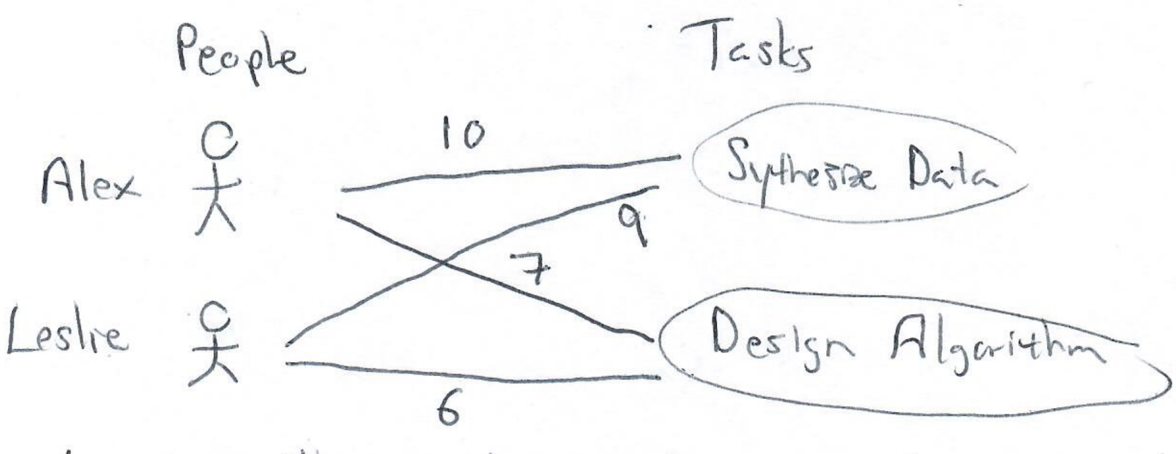
New topics: When is an ILP just a LP?

Examples (Vanderbei, Ch. 14, 15)

(Integer Linear Programming [C,C,Z] Ch 4)

Idea: Some problems (Network Flows, Bipartite Matching) If written as ILP, simplex method for LP relaxation manages to solve the ILP... (Reason: Total Unimod.)

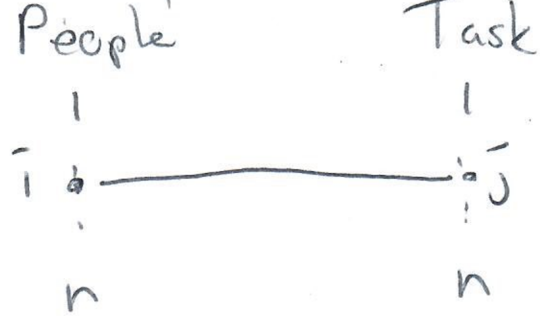
Bipartite Weighted Matching (Assignment Problem)



Want match people one-to-one tasks, max utility.

Here: Alex → Synth Data, Leslie → Designs Alg $10+6=16$ } tie
Alex → Design Alg, Leslie → Synth Data $7+9=16$

Abstractly



weight $= w_{ij}$

utility of matching i 웃 j task

Let $x_{ij} = \begin{cases} 1 & \text{if match } i \leftrightarrow j \\ 0 & \text{otherwise} \end{cases}$

$$\text{maximize} \quad \sum_{i,j} w_{ij} x_{ij} \quad \text{s.t.}$$

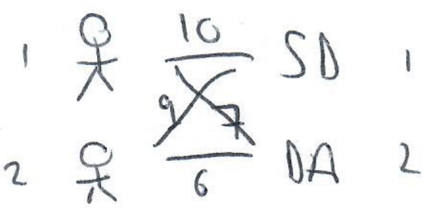
i 웃: $x_{i1} + x_{i2} + \dots + x_{in} = 1$ (i^{th} person does one task)

$x_{1j} + x_{2j} + \dots + x_{nj} = 1$ (j^{th} task done by one person)

and $x_{ij} \geq 0$ and x_{ij} integers (so $x_{ij} = 0, 1$)

---

e.g.



max $10x_{11} + 7x_{12} + 9x_{21} + 6x_{22}$

s.t. $x_{11} + x_{12} = 1$, $x_{11} + x_{21} = 1$

$x_{21} + x_{22} = 1$, $x_{12} + x_{22} = 1$

$x_{11}, x_{12}, x_{21}, x_{22} \geq 0$, ~~($x_{ij}$ integers)~~

relaxation

Optimal

$x_{11} = x_{22} = 1$, $x_{12} = x_{21} = 0$, obj $= 16$

$x_{11} = x_{22} = 0$, $x_{12} = x_{21} = 1$, obj $= 16$

$x_{11} = x_{22} = 7/9$, $x_{12} = x_{21} = 2/9$ obj $= 16$

Why does the simplex find integer solutions?

Short Answer: Total Unimodularity

Longer Answer...

Review Revised Simplex formulas:

$$\max \vec{c}^T \vec{x} \text{ (unimportant) s.t.}$$

$$A\vec{x} \le \vec{b}, \quad \vec{x} \ge 0.$$

$$\rightsquigarrow \vec{x}_{slack} = \vec{b} - A\vec{x}_{dec}$$

$$\underbrace{A\vec{x}_{dec} + \vec{x}_{slack}}_{} = \vec{b}$$

$$\begin{bmatrix} A & \vdots & I \end{bmatrix} \begin{bmatrix} \vec{x}_{dec} \\ \vec{x}_{slack} \end{bmatrix} = \vec{b}$$

Say we reorder $\vec{x}$ components into $\vec{x}_{basic}$, $\vec{x}_{non\text{-}basic}$

$$\begin{bmatrix} A \text{ big} \end{bmatrix} \begin{bmatrix} \vec{x}_{basic} \\ \vec{x}_{non basic} \end{bmatrix} = \vec{b}$$

$$= A_B \vec{x}_B + A_N x_N$$

$$\Rightarrow A_B \vec{x}_B = \vec{b} - A_N \vec{x}_N$$

$$\boxed{\vec{x}_B = A_B^{-1}(\vec{b} - A_N \vec{x}_N)}$$

Claim: For some LP's, some A's, A_B^{-1} will always be integral

To be continued...