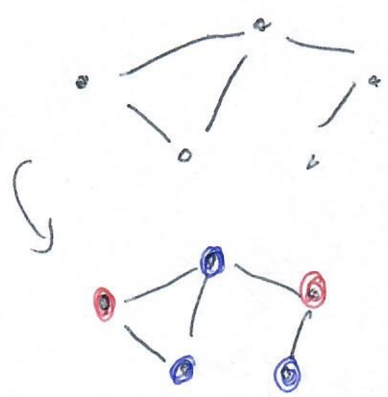
Groups on Friday; if you can't make there will be a piazza.com page

Last time:

Class of ILP: "graph colouring"



$G = (V, E)$

weight : $E \to \mathbb{R}_{\geq 0}$

\# colours

minimize Σ weights of edges that are monochromatic

n vertices $v_1, \dots, v_n$

k colours $\{1, \dots, k\}$

"colouring" map $V \to \{1, \dots, k\}$

<u>edges with endpoints of same colour</u> : take Σ weights.

$$\boxed{\{x_{ij}\}_{\substack{i=1,\dots,n \\ j=1,\dots,k}} = \begin{cases} 1 & \text{if vertex } i \text{ has colour } j \\ 0 & \text{otherwise} \end{cases}}$$

For $i_1, i_2 = 1, \dots, n$ ($i_1 \neq i_2$, $i_1 < i_2$) have weight $w_{i_1 i_2}$

(Problem: Minimize $\sum_{j=1}^{k} \sum_{i_1<i_2} w_{i_1,i_2}$)

Each colour occurs once

$x_{11}+x_{12}+\dots+x_{1k}=1$, $x_{ij}\in\{0,1\}$

Structure:

LP — standard — Application

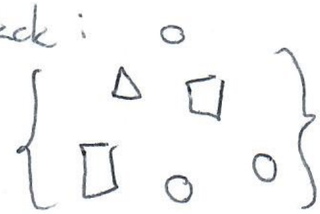
ILP — rest: Bipartite Matching — Apps
Bin Packing — Apps
Vertex Cover
Colouring — Apps

Graph Colouring ⇝ App: Exam scheduling

Bin Packing:

Bins 1 Bin 2 Bin 3

Pack :



Want to pack things into bins

Bin Packing: Rolls of Wrapping Paper

Roll 1: 30 m, Costs \$10
Roll 2: 50 m, Cost \$15

Gifts: gift 1: .5 m, gift 2: 1 m, ..., gift n 2m

Want to pack gifts into rolls at min cost.

e.g.

$g_1, \dots, g_n$ reals

bins $1, \dots, 3$ 30m capacity

bins $4, \dots 7$ 50 m capacity

Can I pack $g_1, \dots, g_n$ into bins? feasible

$g_1 = 28$, $g_2 = 3, \dots$

Gifts $\{1, \dots, n\} \longrightarrow$ Bin $1, \dots 3, 4, \dots 7$

Let $x_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ gift goes into bin } j \\ 0 & \text{otherwise} \end{cases}$

Constraints: For each i,

$x_{i1} + x_{i2} + \dots + x_{i7} = 1$ each gift goes into bins 1 – 7

Bin 1: $x_{11} g_1 + x_{21} g_2 + \dots + x_{n1} g_n \leq 30\,m$

Bin 2: $x_{12} g_2 + \dots + x_{n2} g_n = \sum_{i=1}^{n} x_{i2} g_i \leq 30\,m$

Bin 3: $\sum_{i=1}^{n} x_{i3} g_i \leq 30$ Bin 4: $\sum_{i=1}^{n} x_{i4} g_i \leq 50$, etc.

If some gifts don't fit anywhere

$x_{i1} + x_{i2} + \dots + x_{i7} \leq 1$

maximize $\sum_{i,j} x_{ij} \cdot (\text{value of gift } i)$

$\vdots$