

Sept 25

(1)

- Sign up for gradescope.com accounts
- You need an email address (create one; needs to be functional)
- I don't recommend giving your real name.
- " " " " any ID #.
- Do homework, write reports: ~~you~~ write on assignment your email addresses (gradescope)
- Add math 441
- Please put solutions to different problems on different pages.
- Fill out "UBC Survey" give UBC ID, gradescope email.

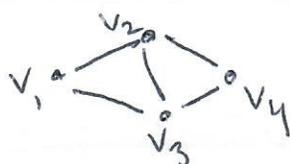
This week: Applications of LP / Integer LP,  
Project ideas, probably Friday or <sup>next</sup> Monday form groups  
Friday Oct 6; or a bit later - first preliminary overview.

Sample project: exam scheduling

- Graph colouring: Graph  $G = (V, E)$ , vertices, edges,



$V$  finite set,  
edge = pair of vertices



$$V = \{v_1, v_2, v_3, v_4\}, E = \left\{ \begin{array}{l} \{v_1, v_2\}, \{v_2, v_3\} \\ \{v_3, v_1\}, \{v_2, v_4\} \\ \{v_3, v_4\} \end{array} \right\}$$

Graph Colouring: We have  $k$  colours, given graph (2)

$G = (V, E)$ . Can we assign colours  $1, \dots, k$  to  $V$  st.

no edge has the same colour?



red, green, blue

each edge has different coloured endpoints

but



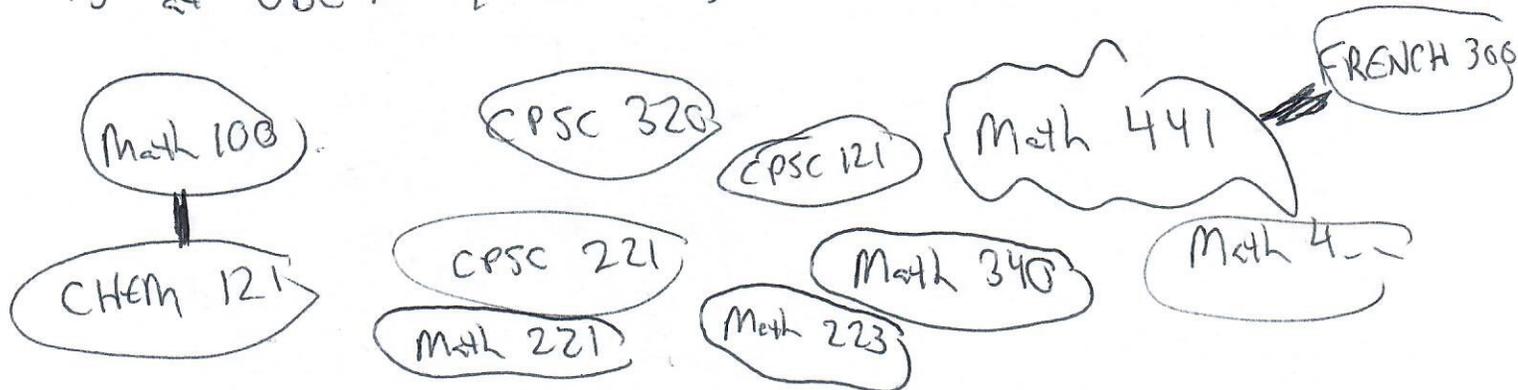
we can't colour the vertices with 3 colours with each edge of endpoints different colours



Application: Schedule exam for classes into exam periods without conflicts:

Graph: Vertices,  $V$ , { set of classes }

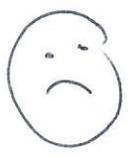
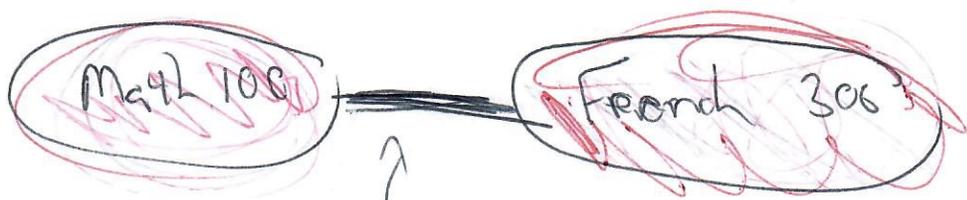
e.g. at UBC: { Math 100, Math 101, Math 102, CPSC 320, ... }



join vertices that share students

Say we have 44 exam periods, say colour each course with its exam period.

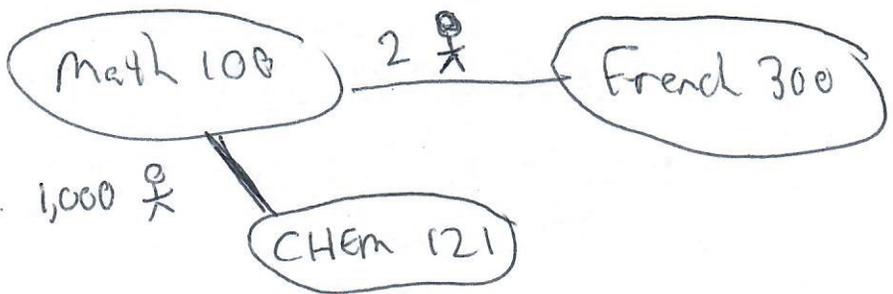
Math 100, French 300 exam period 1 red



they share students, and some colour for endpoints — colour  $\leftrightarrow$  exam period

Asking for a "good colouring", meaning no edge has same two colours,  $\rightsquigarrow$  exam scheduling

Variant: Instead of no conflicts, allow some number of conflicts, or for each edge/conflict give a "weight"



Any graph colouring, weighted or not  $\rightarrow$  solved with ILP

Say  $V = \{v_1, \dots, v_n\}$ , each  $i < j$ ,  $1 \leq i < j \leq n$

we have weight  $w_{ij} = \begin{cases} 0 & \text{if there's no edge } i - j \\ 1 & \text{otherwise, or some positive weight} \end{cases}$

Say colours  $1, \dots, k$

$x_{im} = \begin{cases} 1 & \text{if vertex } i \text{ is assigned colour } m \\ 0 & \text{otherwise} \end{cases}$

Want  $x_{i1} + x_{i2} + \dots + x_{ik} = 1$