Give list of applications of LP & ILP.

Today: - Curve fitting [LP w/o LP]

- Graph colouring & exam scheduling.

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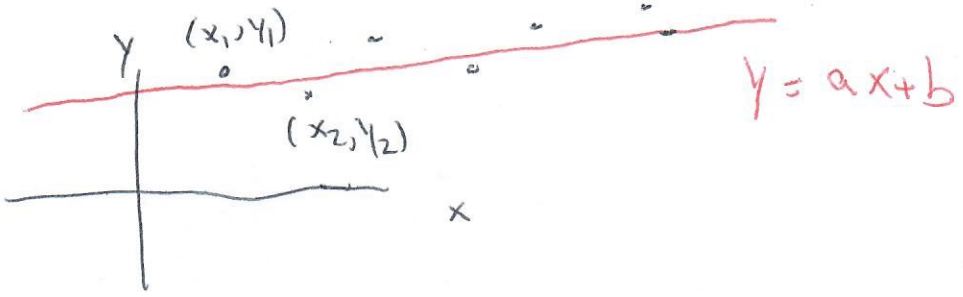
Curve fitting:



Data: $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

Variables (decision variables): a, b

Max or L^∞ fit:

$$\text{MaxError}(a,b) = \max_{i=1,\ldots,n} |y_i - ax_i - b|$$

So $\text{MaxError}(a,b) = 0 \Rightarrow$ you have $y_i = ax_i + b$ for all $i = 1, \ldots, n$. Exact.

Problem: find a, b s.t.

$$\text{MaxError}(a,b) = \max |y_i - ax_i - b|$$

is minimized.

If the max error for a, b is w, then

$$\begin{aligned} -w &\le y_1 - ax_1 - b \le w \\ -w &\le y_2 - ax_2 - b \le w \\ &\vdots \end{aligned}$$

So really want

minimize w s.t.

i.e.

$$\begin{aligned} y_1 - ax_1 - b &\le w \\ -y_1 + ax_1 + b &\le w \\ y_2 - ax_2 - b &\le w \\ -y_2 + ax_2 + b &\le w \\ &\vdots \end{aligned}$$

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e.g. $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (2, 5)$, ——

how well does $y = ax + b$ fit $(1, 3)$ means

$\Leftrightarrow$ what is $|3 - a - b|$

$\Leftrightarrow$ what is smallest w with

$$-w \le 3 - a - b \le w$$

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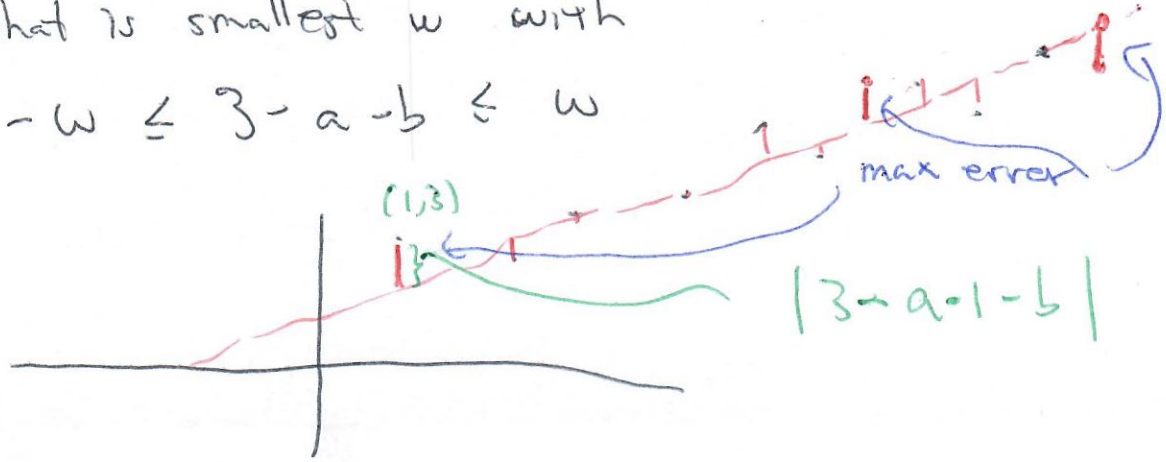
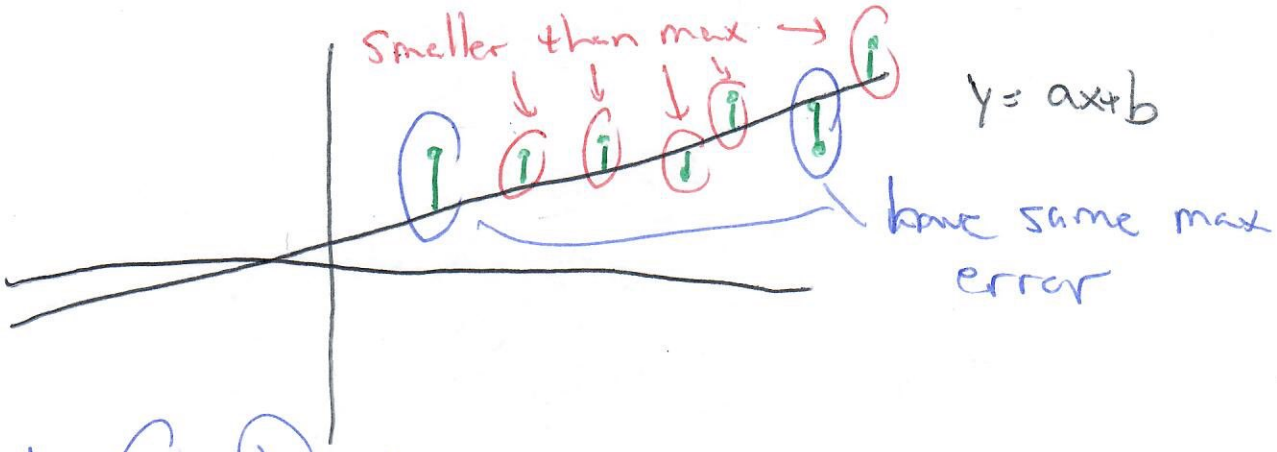
Guess :



Curve fitting to max error ⇝ LP

Observed that there are "3 support points" where max error occurs. Why?

- The max errors at one point.
- Could it occur at two points?



$y = (ax) + (b)$ two parameters ⇝ 3 support points

- ⇝ liner, 3 parameters ⇝ 4 support points

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Linear Prog w/o Linear Prog decision: a, b, w

$$y_1 - ax_1 - b \le w \quad \leftarrow s_1 \text{ slack}$$
$$-y_1 + ax_1 + b \le w \quad \leftarrow s_2$$
$$y_2 - ax_2 - b \le w \quad \leftarrow s_3$$
$$-y_2 + ax_2 + b \le w \quad \leftarrow s_4$$
$$\vdots \qquad\qquad \vdots$$

Decision w, a, b Slack $s_1, \ldots, s_{2n}$

$$s_1 = w - (y_1 - ax_1 - b)$$

s_1 is 0 then worst possible fit, i.e a support point

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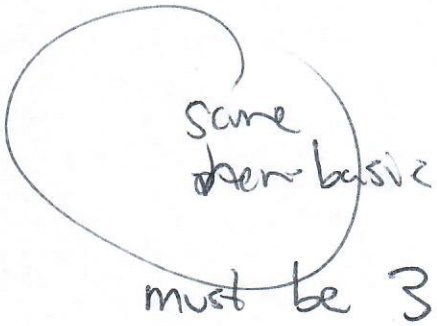
Assume fit isn't perfect, so $w > 0$;

" $a, b > 0$

Simplex method steps:



must be 3,
since started with
a, b, w as decision
vars

$\Rightarrow$ in end, since $a, b, w > 0$,
we have 3 slack = 0 in optimal sol