Branch & Bound

Say LP $\quad \max \vec{c}^T \vec{x}$ s.t. $A\vec{x} \le \vec{b}$, $\vec{x} \ge 0$

- Usually $\vec{x}$ varies over the reals;
- the ILP (integer LP) means $\vec{x} \in \mathbb{Z}^n$,

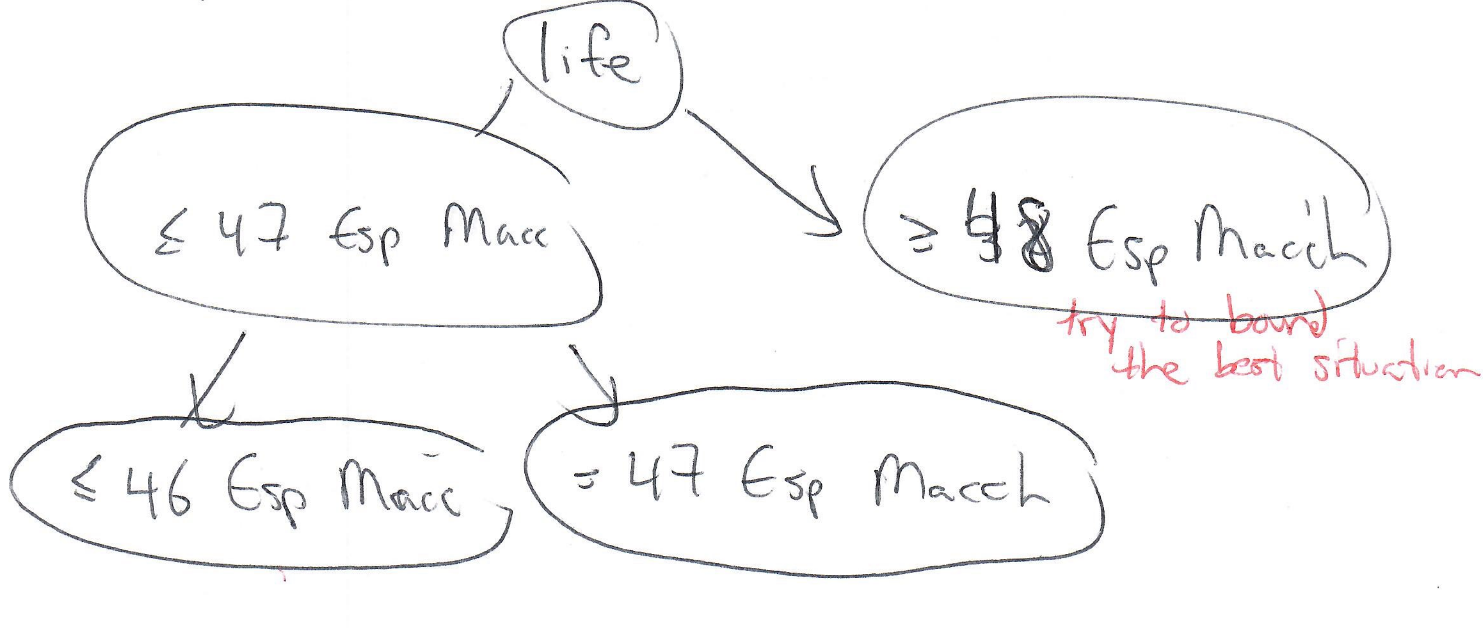
  x has integer coefficients

- say that we think $x_1 = 31$ Lattés

  $x_2 = 47$ Esp. Macch

  maybe best

  is optimal:



Examples: max $4x_1 + 5x_2$ s.t.

	first	second (Easy)	third (Trickier)
$x_1 + 2x_2 \le$	8	18.07	
$x_1 + x_2 \le$	5	11.05	
$2x_1 + x_2 \le$	8	18	
$x_1, x_2 \ge 0$			

First: Say x_1, x_2 have to be integers.

- Solved the LP where x_1, x_2 any reals.
- This is called the <u>relaxation</u>.
- Found: LP optimized at $x_1 = 2$, $x_2 = 3$. Done!

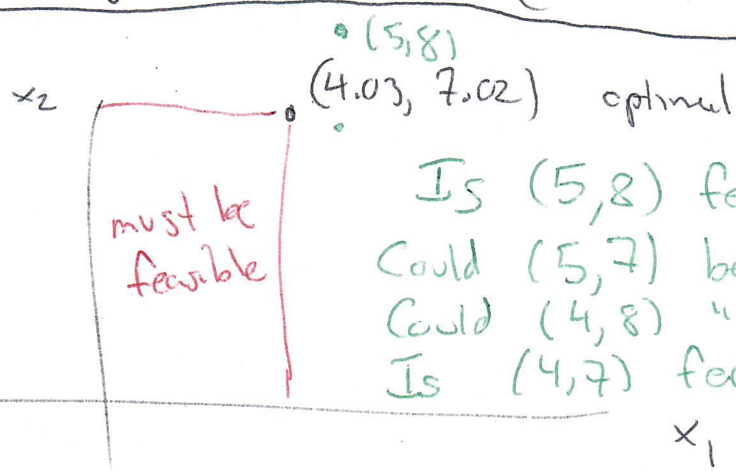
Second: Constraints 18.07, 11.05, 18

- LP optimized at $x_1 = 4.03$, $x_2 = 7.02$

Easy LP: max $\vec{c}^T \vec{x}$ s.t. $A\vec{x} \le \vec{b}$, $\vec{x} \ge 0$

Standard production from resources: $A, \vec{b}, \vec{c}$
non-neg components/entries.

- We want x_1, x_2 to be integers.
- Solved LP, found optimum:
  - Esattes $= x_1 = 4.03$
    Esp M. $= x_2 = 7.02$
    objective $z = z(4.03, 7.02) = 51.22$
- However $x_1 = 4$, $x_2 = 7$ feasible, $x_1, x_2 \in \mathbb{Z}$ (integers)
  obj $(4,7) = 51$ (close to 51.22)



Is (5,8) feasible? No!!!
Could (5,7) be feasible? Maybe!!
Could (4,8) " " ? Maybe!!
Is (4,7) feasible? Yes!!

Branch! Esp ≥ 8 ...
Esp ≤ 7