- LP without LP
- Branch & Bound

TODAY

1. LP w/o LP ← Use that fact that simplex method work...
2. Branch & Bound

Summarized last time's example

(y_1) same $\rightarrow x_1 + 2x_2 \leq 8$

(y_2) same $\rightarrow x_1 + x_2 \leq 5$

(y_3) same $\rightarrow 2x_1 + x_2 \leq 8$

$\max (4x_1 + 5x_2) = z = z(\vec{x})$

$x_1, x_2 \geq 0$

add to get upper bound on $4x_1 + 5x_2$

① $5(x_1 + x_2 \leq 5)$

$4x_1 + 5x_2 \leq 5x_1 + 5x_2 \leq 25 \Rightarrow z(\vec{x}) \leq 25$

② 3(first ineq) + 1(second) → better

$y_1 (x_1 + 2x_2 \leq 8)$

$y_2 (x_1 + x_2 \leq 5)$

$y_3 (2x_1 + x_2 \leq 8)$

$(y_1, y_2, y_3 \geq 0)$

$(y_1 + y_2 + 2y_3)x_1 + (2y_1 + y_2 + y_3)x_2 \leq 8y_1 + 5y_2 + 8y_3$

So if: $(y_1 + y_2 + 2y_3)$ at least 4; $(2y_1 + y_2 + y_3)$ at least 5; $8y_1 + 5y_2 + 8y_3$ bound on LP

Dual LP: minimize $8y_1 + 5y_2 + 8y_3$

s.t. $y_1 + y_2 + 2y_3 \geq 4$, $2y_1 + y_2 + y_3 \geq 5$

row $\equiv$ $A\vec{x} \le \vec{b}$, $\vec{x} \ge 0$

$[y_1\ y_2\ y_3]\left(A\vec{x} \le \vec{b}\right)$

$\vec{y}^T\left(A\vec{x} \le \vec{b}\right)$ (if $\vec{y} \ge 0$)

$\Longrightarrow$ $\left(\vec{y}^T A\right)\vec{x} \le \vec{y}^T b$

$$\max 4x_1 + 5x_2 \quad \text{s.t.}$$
$$x_1 + 2x_2 \leq 8$$
$$x_1 + x_2 \leq 6 \quad \text{(more coffee :)}$$
$$2x_1 + x_2 \leq 8$$

but x_1, x_2 have to be integers...

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Branch & bound:

$x_1 + x_2 \leq 6$ means $x_1 = 0, 1, \ldots, 6$, $x_2 = 0, 1, \ldots, 6$

could try out $7 \cdot 7 = 49$ possibilities