

$$\max 4x_1 + 5x_2 \quad \text{s.t.}$$

$$x_1 + 2x_2 \leq 8$$

$$x_1 + x_2 \leq 5$$

$$2x_1 + x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

Conceptually $\max \vec{c} \cdot \vec{x}_{\text{dec}}$

$$A \vec{x}_{\text{dec}} \leq \vec{b}$$

$$\vec{x}_{\text{dec}} \geq 0$$

$$\vec{x}_{\text{slack}} = \vec{b} - A \vec{x}_{\text{dec}}$$

Concretely

$$x_3 = 8 - x_1 - 2x_2$$

$$x_4 = 5 - x_1 - x_2$$

$$x_5 = 8 - 2x_1 - x_2$$

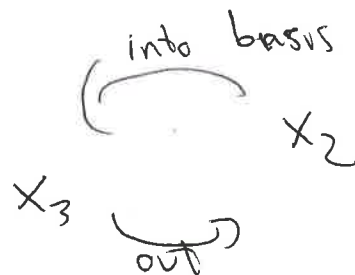
$$z = 4x_1 + 5x_2$$

hold fixed

increase

x_3 hits zero } at $x_2 = 4$
 $x_4, x_5 \geq 0$

pivot



- Simplex Method

- Duality

- Revised Simplex Formulas

- Sensitivity Analysis

- NEW!

Simplex Method without

the " " "

Feasible Solution

non-basic $\rightarrow 0$

$x_1, x_2 \rightarrow 0$

$x_3 = 8, x_4 = 5, x_5 = 8$

Sensitivity Analysis

$$\begin{array}{l}
 x_1 + 2x_2 \leq 8 \quad \leftarrow \text{acidity} \\
 x_1 + x_2 \leq 5 \quad \leftarrow \text{coffee} \\
 2x_1 + x_2 \leq 8 \quad \leftarrow \text{milk}
 \end{array}$$

$\leftarrow \text{changes } x_3$
 $\leftarrow x_4$

\uparrow \uparrow
 Lattes Esp Macs

Objective $z = 4x_1 + 5x_2$

What happens if $z = 4x_1 + 5x_2 \rightarrow 4x_1 + 5.2x_2$

Or $x_1 + x_2 \leq 5 \rightarrow x_1 + x_2 \leq 5.1$

e.g. final dictionary

utility $\rightarrow z = 23 - 3x_4 - 1x_3$ coffee

Say coffee gets more expensive

$x_1 + x_2 \leq 5 \rightarrow x_1 + x_2 \leq 4.3$

We used to be optimal at $(x_1, x_2) = (2, 3), z = 23$

Milk = $x_5 = ?$ $3 = 3$ utility / coffee utility / acidity

$\rightarrow 2x_1 + x_2 \leq 8$ currently $2x_1 + x_2 = 7$ at optimality

Duality: $\max 4x_1 + 5x_2$ s.t.

$$1 \quad (x_1 + 2x_2 \leq 8)$$

$$3 \quad (x_1 + x_2 \leq 5)$$

$$2x_1 + x_2 \leq 8 \quad (x_1, x_2 \geq 0)$$

Say we notice $x_1=2, x_2=3$ is feasible, $z=23$

Notice

$$x_1 + x_2 \leq 5 \Rightarrow 5x_1 + 5x_2 \leq 25$$

$$z = 4x_1 + 5x_2 \text{ and } x_1 \geq 0$$

$$\leq 5x_1 + 5x_2 \leq 25 \text{ by}$$

Can we do better?

1 first + 3 second

$$x_1 + 2x_2 \leq 8$$

$$3x_1 + 3x_2 \leq 15$$

$$4x_1 + 5x_2 \leq 23$$

!!

$$z = 23 - 3x_1 - x_3$$

$$y_1 (x_1 + 2x_2 \leq 8)$$

$$y_2 (x_1 + x_2 \leq 5)$$

$$y_3 (2x_1 + x_2 \leq 8)$$

bound on

$$z = 4x_1 + 5x_2$$