$$\max\ 4x_1 + 5x_2 \quad \text{s.t.}$$

$$x_1 + 2x_2 \le 8$$

$$x_1 + x_2 \le 5$$

$$2x_1 + x_2 \le 8$$

$$x_1, x_2 \ge 0$$

- Simplex Method
- Duality
- Revised Simplex formulas
- Sensitivity Analysis
- NEW!

Simplex Method without the " "

Conceptually $\max\ \vec{c} \cdot \vec{x}_{dec}$

$$A\vec{x}_{dec} \le \vec{b}$$

$$\vec{x}_{dec} \ge 0$$

$$\vec{x}_{slack} = \vec{b} - A\vec{x}_{dec}$$

Concretely

$$\boxed{x_3 = 8 - x_1 - 2x_2}$$

$$x_4 = 5 - x_1 - x_2$$

$$x_5 = 8 - 2x_1 - x_2$$

$$z = 4x_1 + 5x_2$$

hold fixed ($4x_1$) increase ($5x_2$)

Feasible Solution

non-basic $\to 0$

$x_1, x_2 \to 0$

$x_3 = 8,\ x_4 = 5,\ x_5 = 8$

$\left.\begin{array}{l} x_3 \text{ hits zero} \\ x_4, x_5 \ge 0 \end{array}\right\}$ at $x_2 = 4$

pivot x_3 out, x_2 into basis

Sensitivity Analysis

$x_1 + 2x_2 \le 8 \leftarrow$ acidity $\leftarrow$ changes $\leftarrow x_3$

$x_1 + x_2 \le 5 \leftarrow$ ~~mocha~~ coffee $\leftarrow x_4$

$2x_1 + x_2 \le 8 \leftarrow$ milk

$\uparrow$ Lattes $\quad \uparrow$ Esp Mac.

Objective $\quad z = 4x_1 + 5x_2$

What happens if $\quad z = 4x_1 + 5x_2 \to 4x_1 + 5.2x_2$

Or $\quad x_1 + x_2 \le 5 \to x_1 + x_2 \le 5.1$

e.g. final dictionary

utility $\to z = 23 - 3x_4 - 1x_3$ ~~mocha~~ coffee

Say coffee ~~gets~~ more expensive

$x_1 + x_2 \le 5 \to x_1 + x_2 \le 4.3$

We used to be optimal at $(x_1, x_2) = (2, 3)$, $z = 23$

$3 = 3$ utility / coffee $\qquad$ 1 utility / acidity

Milk $= x_5 = ???$

$2x_1 + x_2 \le 8$ currently $\qquad 2x_1 + x_2 = 7$ at optimality

Duality: $\max\ 4x_1 + 5x_2$ s.t.

1 $(x_1 + 2x_2 \le 8)$

3 $(x_1 + x_2 \le 5)$

$2x_1 + x_2 \le 8$ $(x_1, x_2 \ge 0)$

Say we notice $x_1 = 2,\ x_2 = 3$ is feasible, $\underline{z} = 23$

Notice $x_1 + x_2 \le 5 \Rightarrow 5x_1 + 5x_2 \le 25$

$z = 4x_1 + 5x_2$ and $x_1 \ge 0$

$\le 5x_1 + 5x_2 \le 25$ by

Can we do better?

1 first + 3 second

$$x_1 + 2x_2 \le 8$$
$$3x_1 + 3x_2 \le 15$$
$$\overline{4x_1 + 5x_2 \le 23}$$!!

$z = 23 - 3x_4 - 1x_3$

$y_1 \quad (x_1 + 2x_2 \leq 8)$

$y_2 \quad (x_1 + x_2 \leq 5)$

$y_3 \quad (2x_1 + x_2 \leq 8)$

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bound on $z = 4x_1 + 5x_2$