- Announcements:
  - Pick groups in 2-3 weeks after we have a lot of models
  - Rem conditions ←
  - Recommend Gurobi - easy
    - almost best
    - use anything you want
  - I will use Maple to investigate what's going on with optimization problems

NOW - We review models in LP
cover all Math 340 in each

$\max \vec{c} \cdot \vec{x}$

$A\vec{x} \underset{\text{dec}}{\leq} \vec{b}$

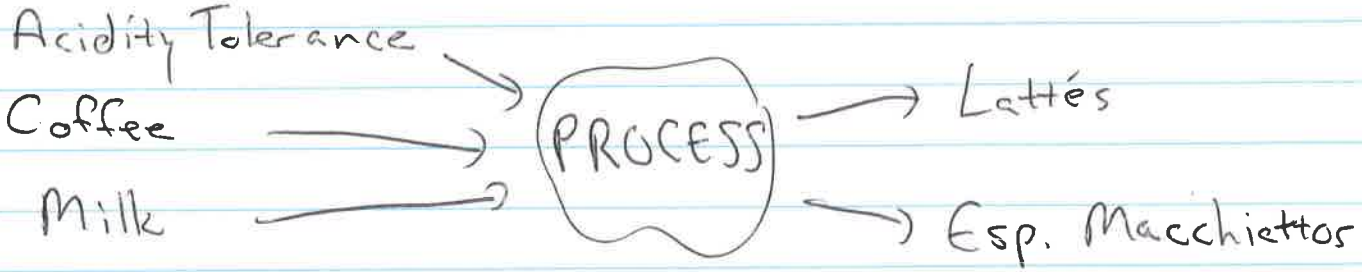
$\vec{x} \geq 0$

$\max \; 4x_1 + 5x_2$ s.t.

$x_1 + 2x_2 \leq 8$ Acidity

$x_1 + x_2 \leq 5$ Coffee

$2x_1 + x_2 \leq 8$ Milk



{Resources / Inputs} → PROCESS → {Products / Outputs}

Special Properties:

① $\vec{b} \geq \vec{0} \Rightarrow \vec{x} = \vec{0}$ is feasible

② A's entries $\geq 0 \Rightarrow$ If $\vec{x}$ is feasible and $\vec{0} \leq \vec{x}' \leq \vec{x}$

$\Rightarrow \vec{x}'$ is feasible

Problem: max $z = \vec{c} \cdot \vec{x}$ s.t. $A\vec{x} \leq \vec{b}$, $\vec{x} \geq 0$ | Math 441 @ 0911

Simplex Method:

$$\vec{x}_{slack} = \vec{b} - A\vec{x}_{dec}$$

$$z = \vec{c} \cdot \vec{x}_{dec}$$

$$x_3 = 8 - x_1 - 2x_2$$

$$x_4 = 5 - x_1 - x_2$$

$$x_5 = 8 - 2x_1 - x_2$$

$$z = 4x_1 + 5x_2$$

View: Initial Dictionary, Intial BFS: $x_1 = x_2 = 0$

x_3, x_4, x_5 given

$z = 0 + 4x_1 + 5x_2 \Rightarrow$ increase z by increasing either x_1, x_2

hold x_1 at 0

x_2 increase

max $z = 4x_1 + 5x_2$ s.t.

$x_1 + 2x_2 \le 8$ ↪ $x_3 = 8 - x_1 - 2x_2 \ge 0$

$x_1 + x_2 \le 5$ $x_4 = 5 - x_1 - x_2$

$2x_1 + x_2 \le 8$ $x_5 = 8 - 2x_1 - x_2$

dictionary

$$x_1 + 2x_2 + x_3 = 8$$
$$x_1 + x_2 + x_4 = 5$$
$$2x_1 + x_2 + x_5 = 8$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix} = \vec{b}$$

x_{dec} = x_1, x_2; x_{slack} = x_3, x_4, x_5

A_{orig} | I

A_{big}

Possible 1st Pivot

Hold x_1 fixed, increase x_2:

$$x_3 = 8 - 0 - 2x_2 \geq 0 \qquad 2x_2 \leq 8$$

$$x_4 = 5 - 0 - x_2 \geq 0 \qquad x_2 \leq 5$$

$$x_5 = 8 - x_2 \geq 0 \qquad x_2 \leq 8$$

$$x_2 \leq 4$$

When $x_2 \to 4$, $x_3 \to 0$, $x_4 \to 1$, $x_5 \to 4$ } x_3 leaves

$$x_3 = 8 - x_1 - 2x_2 \qquad (x_2 \to 4)$$

$$\Leftrightarrow \quad 2x_2 = 8 - x_1 - x_3$$

$$x_2 = 4 - \tfrac{1}{2}x_1 - \tfrac{1}{2}x_3 \quad \text{(MAGIC!)}$$

4 1/2

$x_3 = 8 - x_1 - 2x_2$

$x_4 = 5 - x_1 - x_2$

$x_5 = 8 - 2x_1 - x_2$

$z = 4x_1 + 5x_2$

x_2 enters, x_3 leaves

$x_2 = 4 - \frac{1}{2}x_1 - \frac{1}{2}x_3$

$x_4 = 5 - x_1 - (4 - \frac{1}{2}x_1 - \frac{1}{2}x_3)$

$= 1 - \frac{1}{2}x_1 + \frac{1}{2}x_3$

$x_5 = 8 - 2x_1 - (4 - \frac{1}{2}x_1 - \frac{1}{2}x_3)$

$= 4 - \frac{3}{2}x_1 + \frac{1}{2}x_3$

$z = 4x_1 + 5(4 - \frac{1}{2}x_1 - \frac{1}{2}x_3)$

$= 20 + \frac{3}{2}x_1 - \frac{5}{2}x_3$

$$z = 20 + \frac{3}{2}x_1 - \frac{5}{2}x_3 \quad \leftarrow x_1 \text{ must enter}$$

$$x_2 = 4 - \frac{1}{2}x_1 - \frac{1}{2}x_3 \qquad \frac{1}{2}x_1 \le 4$$

$$x_4 = 1 - \frac{1}{2}x_1 + \frac{1}{2}x_3 \qquad \frac{1}{2}x_1 \le 1$$

$$x_5 = 4 - \frac{3}{2}x_1 + \frac{1}{2}x_3 \qquad \frac{3}{2}x_1 \le 4$$

~~BASIS~~ NON-BASIC

$x_1 \rightarrow 2$ x_4 leaves

$x_4 \rightarrow 0$

$x_5 \rightarrow 1$

$x_2 \rightarrow 3$

x_3 stays 0 always

Step 2

$$\frac{1}{2}x_1 = 1 - x_4 + \frac{1}{2}x_3$$

$$x_1 = 2 - 2x_4 + x_3$$

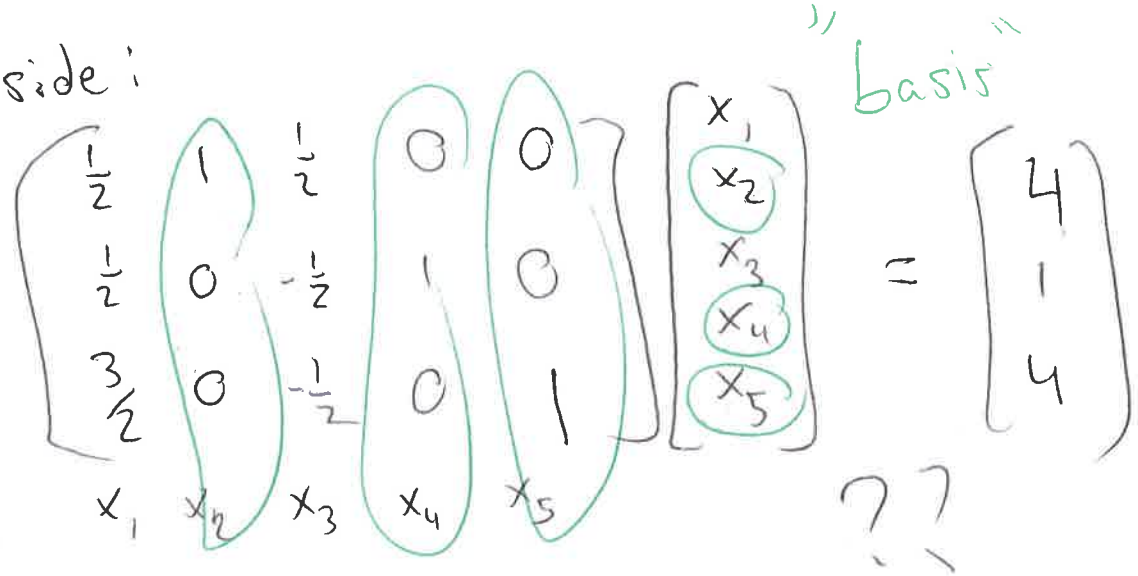
This was to the right of slide (5)

Still $x_1 = 0$, $x_3 = 0$ $\longrightarrow$ $z = 20$, $x_2 = 4$, $x_4 = 1$, $x_5 = 4$

is a valid solution to $A\vec{x} \leq \vec{b}$

Feasible!

Aside:



$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix}$$

Is this the same information??

STILL TO COME (1)

$$z = 20 + \frac{3}{2} x_1 - \frac{5}{2} x_3$$

$$= 20 + \frac{3}{2}(2 - 2x_4 + x_3) - \frac{5}{2} x_3$$

$$= 23 - 3x_4 - x_3$$

STOP!

$x_2 = 3 +$ unimportant

$x_1 = 2 +$ unimportant

$x_5 = 1 +$ unimportant

$z = 23 - 3x_4 - x_3$

STILL TO COME (2)

$$z = 23 - 3x_4 - x_3$$

objective is "study power"

x_3 = acidity

x_4 = coffee

x_5 = milk

3 = study power / coffee

1 = study pwr / acidity

=

Q: What if milk resource changes a bit?