HOMEWORK #1, MATH 441, FALL 2017

JOEL FRIEDMAN

(1) Consider the linear program max $\vec{c}^T\vec{x}$ subject to $A\vec{x} \le \vec{b}$ and $\vec{x} \ge 0$, where

$$\vec{c} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 25 \\ 7 \\ 8 \end{bmatrix}.$$

You run the simplex method on this LP and obtain the final dictionary:

$$\begin{aligned} x_1 &= 1 + x_4 - x_5 \\ x_2 &= 6 - 2x_4 + x_5 \\ x_3 &= 6 + 5x_4 - 2x_5 \\ z &= 22 - 2x_4 - x_5 \end{aligned}$$

(a) Put this into an LP optimization software, and verify that your software works on this example. Print out your LP description, and the output of the software; for example, if you use Gurobi, then print out the file describing the LP and the optimal solution Gurobi finds as well as the values of the x_1, x_2.

Solution: See gurobi files at the end.

(b) Change the constraint $x_1 + x_2 \le 7$ to $x_1 + x_2 \le 7.01$, and run your optimization software again. What is the new optimum z value and optimum solution (x_1, x_2)?

Solution: $z = 22.02$, $x_1 = 0.99$, $x_2 = 6.02$.

How could you have predicted this from the dictionary?

Solution: Since x_4 is the slack variable corresponding to the inequality $x_1 + x_2 \le 7$, the change of 0.01 increases the objective z by 0.02, corresponding to the coefficient of -2 in the optimal dictionary line $z = 22 - 2x_4 - x_5$; similarly the x_4 coefficients of

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1 and -2 in this dictionary for x_1 and x_2 (respectively) account for the change of -0.01 and 0.02 in these variables new values in the optimal solution.

(c) Same question where the constraint changes to $x_1 + x_2 \leq 6.99$.

Solution: When we run our software we get $z = 21.98$, $x_1 = 1.01$, $x_2 = 5.98$. Now we have that the change in the x_4 constraint is -0.01, which is why we see the changes in z, x_1, x_2 of, respectively, $-0.02, 0.01, -0.02$.

(d) Same question where you leave $x_1 + x_2 \leq 7$, but now change the first constraint to $x_1 + 3x_2 \leq 25.01$.

Solution: Your software should show the same optimum solution as in part (a). This is because the change of 25 to 25.01 corresponds to a change in x_3, which is a constraint that is not active in part (a), i.e., that is satisfied with strict inequality.

(2) Use your software to solve the linear program: maximize x_1 subject to $x_1 \geq 4$, $x_1 \leq 3$, and $x_1 \geq 0$. Print its output and make sure that your software says that the above linear program is infeasible.

Solution: See the Gurobi file and output for this exercise.

Department of Computer Science, University of British Columbia, Vancouver, BC V6T 1Z4, CANADA, and Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, CANADA.

E-mail address: jf@cs.ubc.ca or jf@math.ubc.ca

URL: http://www.math.ubc.ca/~jf