HOMEWORK #6, MATH 441, FALL 2017

JOEL FRIEDMAN

Please note:

(1) You may work together on homework, but you must write up your own solutions individually. In particular, you must write your own code, spreadsheets, etc.
(2) You must acknowledge with whom you worked (specify their `gradescope.com` email addresses). You must also acknowledge any sources you have used beyond the textbook and class material.
(3) When you submit your homework to `gradescope.com`, you need to put the solutions to different problems on different pages; `gradescope.com` will ask you to identify which pages correspond to which problems.

(1) Let $\mu > 0$ be a real parameter, and consider the problem of minimizing $f(w_1, w_2)$ subject to $g_i(w_1, w_2) \le 0$ for $i = 1, \dots, 4$, where

$$f(w_1, w_2) = 100 - 4\mu(w_1^2 - 2w_1w_2 + w_2^2), \quad g_1(w_1, w_2) = w_1 + w_2 - 10$$

$$g_2(w_1, w_2) = -w_1 - w_2 + 10, \quad g_3(w_1, w_2) = -w_1, \quad g_4(w_1, w_2) = -w_2.$$

(Note the similarity to Problem 2 from Homework 4.) Answer the following questions and justify your answer:

(a) Describe the feasible region of this program as a subset of $(w_1, w_2) \in \mathbb{R}^2$.
(b) For each feasible (w_1, w_2), describe which of the $g_i \le 0$ are active constraints. [You may draw a diagram or make a list for each subset of $i = 1, 2, 3, 4$, but you should justify your answer in words either way.]
(c) Find all KKT points of this program.
(d) Relate your findings to the solution of Problem 2 of Homework 4.

Department of Computer Science, University of British Columbia, Vancouver, BC V6T 1Z4, CANADA, and Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, CANADA.

E-mail address: `jf@cs.ubc.ca or jf@math.ubc.ca`

URL: `http://www.math.ubc.ca/~jf`

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