HOMEWORK #5, MATH 441, FALL 2017

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Please note:

(1) You may work together on homework, but you must write up your own solutions individually. In particular, you must write your own code, spreadsheets, etc.
(2) You must acknowledge with whom you worked (specify their `gradescope.com` email addresses). You must also acknowledge any sources you have used beyond the textbook and class material.
(3) When you submit your homework to `gradescope.com`, you need to put the solutions to different problems on different pages; `gradescope.com` will ask you to identify which pages correspond to which problems.

(1) Consider our process of scheduling group presentations for MATH 441 this term (see the course website, specifically the webpage on the schedule of presentations). Recall that we formed a utility function based on group preferences for the class day of their presentation; each group presents on exactly one day, and at most three groups can present on any one day. Recall that if group Z expressed preferences 456123987 (so that day 4 is their first choice, day 5 their second choice, and day 7 their last choice), we added the term

$$9Z_4 + 8Z_5 + 7Z_6 + 6Z_1 + 5Z_2 + 4Z_3 + 3Z_9 + 2Z_8 + Z_7$$

to the utility function. Then we used Gurobi to find a utility maximizing schedule.

Assume that there are 16 groups and 9 days over which we schedule them, and that each group has expressed their preferences as above. **Justify your answers** (a simple "yes" or "no" will give you no credit) to the questions below regarding any possible optimal schedule, i.e., utility maximizing schedule, that Gurobi finds.

(a) Say that each group has the same preferences, say 123456789. Can each group get its first choice in an optimal schedule? Could some group get its last choice in an optimal schedule? Is there a unique optimal schedule in this case?

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(b) Under any set of preferences, can a group be assigned its last choice in an optimal schedule?
(c) Under any set of preferences and in any optimal schedule, is it possible that groups A and B would rather switch days? (In other words, group A prefers the day assigned to B than the day assigned to them, and group B prefers the day assigned to group A than the day assigned to them.)
(d) Under any set of preferences and in any optimal schedule, is it possible that group A would rather switch days with group B but that group B does not want to switch with group A? If your answer is "yes," give an example of a set of preferences and an optimal solution where this would happen.
(e) Under any set of preferences and in any optimal schedule, is it possible that group A would rather switch their day to some day that is not fully booked?
(f) Under any set of preferences and in any optimal schedule, is it possible that group A would rather have group B's day, and group B would rather have group C's day, and group C would rather have group A's day?

(2) Which statement best characterizes the utility function in Problem 1:
(a) The utility function is a precise measurement of the overall benefit to society in measurable form; for example, people will be nine times wealthier if each group is assigned its first choice (if possible) than if each group is assigned its last choice (if possible).
(b) The utility function is used because its optimal solutions have certain desirable properties.

Justify your answer in 25 words or fewer.

(3) Which statement best characterizes the Markowitz utility function:
(a) The utility function is a precise measurement of the overall benefit to society in some measurable form.
(b) The utility function is used because its optimal solutions have certain desirable properties.

Justify your answer succinctly.

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