HOMEWORK #2, MATH 441, FALL 2017

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Please note:

(1) You may work together on homework, but you must write up your own solutions individually. In particular, you must write your own code, spreadsheets, etc.
(2) You must acknowledge with whom you worked (specify their `gradescope.com` email addresses). You must also acknowledge any sources you have used beyond the textbook and class material.
(3) When you submit your homework to `gradescope.com`, you need to put the solutions to different problems on different pages; `gradescope.com` will ask you to identify which pages correspond to which problems.

(1) Use branch and bound to solve the integer linear program max $\vec{c}^T\vec{x}$ subject to $A\vec{x} \le \vec{b}$ and $\vec{x} \ge 0$ and $\vec{x} \in \mathbb{Z}^2$ (i.e., x_1, x_2 must be integers)

$$\vec{c} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 29.8 \\ 7.3 \\ 8.2 \end{bmatrix}.$$

Do not make use of the specific properties of $A, \vec{b}, \vec{c}$ in this problem (i.e., that they all have non-negative entries/coefficients). Specifically:

(a) Enter the corresponding LP into your LP software; you should find that the optimum solution is $x_1 = 0.9$, $x_2 = 6.4$, and $z = 22.8$.
(b) Try the following branches: $x_2 \le 6$ and $x_2 \ge 7$. If you need to explore the $x_2 \le 6$ branch further, divide this branch into $x_2 \le 5$ and $x_2 = 6$; if you need to explore the $x_2 \ge 7$ branch further, divide this branch into $x_2 = 7$ and $x_2 \ge 8$. (You should find that the branch $x_2 \ge 8$ is infeasible.)
(c) When you reach a branch with x_2 fixed, branch on x_1 in a similar fashion (solve the relaxed LP, and round up and down).
(d) Complete the branch and bound, and make a diagram of the result.

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(2) Try the above branch and bound method on the integer program with

$$\vec{c} = \begin{bmatrix} 1 \\ 500 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 100 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2030 \end{bmatrix}.$$

Specifically, try branch and bound by searching the possible values of x_2 based on the LP relaxation, branching on x_1 values on branches where the x_2 value has been fixed. Then do the same where you first branch on x_1 values, then x_2 values. Is there a significant difference? Explain.

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