HOMEWORK #1, MATH 441, FALL 2017

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Please note:

(1) You may work together on homework, but you must write up your own solutions individually. In particular, you must write your own code, spreadsheets, etc.
(2) You must acknowledge with whom you worked (specify their `gradescope.com` email addresses). You must also acknowledge any sources you have used beyond the textbook and class material.
(3) When you submit your homework to `gradescope.com`, you need to put the solutions to different problems on different pages; `gradescope.com` will ask you to identify which pages correspond to which problems.

The main point of this homework is to make sure you have access to some sort of LP solving software. I recommend using Gurobi, but you can use any software that works; in the first few homeworks you will be able to check that your software works.

(1) Consider the linear program max $\vec{c}^T\vec{x}$ subject to $A\vec{x} \le \vec{b}$ and $\vec{x} \ge 0$, where

$$\vec{c} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 25 \\ 7 \\ 8 \end{bmatrix}.$$

You run the simplex method on this LP and obtain the final dictionary:

$$\begin{aligned} x_1 &= 1 + x_4 - x_5 \\ x_2 &= 6 - 2x_4 + x_5 \\ x_3 &= 6 + 5x_4 - 2x_5 \\ z &= 22 - 2x_4 - x_5 \end{aligned}$$

(a) Put this into an LP optimization software, and verify that your software works on this example. Print out your LP description, and the output of the software; for example, if you use Gurobi, then print out

Research supported in part by an NSERC grant.

the file describing the LP and the optimal solution Gurobi finds as well as the values of the x_1, x_2.

(b) Change the constraint $x_1 + x_2 \leq 7$ to $x_1 + x_2 \leq 7.01$, and run your optimization software again. What is the new optimum z value and optimum solution (x_1, x_2)? How could you have predicted this from the dictionary?

(c) Same question where the constraint changes to $x_1 + x_2 \leq 6.99$.

(d) Same question where you leave $x_1 + x_2 \leq 7$, but now change the first constraint to $x_1 + 3x_2 \leq 25.01$.

(2) Use your software to solve the linear program: maximize x_1 subject to $x_1 \geq 4$, $x_1 \leq 3$, and $x_1 \geq 0$. Print its output and make sure that your software says that the above linear program is infeasible.

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