Math 340–102 Fall 1999

Midterm

[40 points total]

[12 points] 1. Use the two-phase method to solve

$$\text{maximize} \quad x_1 + 2x_2, \quad \text{subject to } x_1, x_2 \geq 0 \text{ and}$$
$$3x_1 + 2x_2 \leq 6$$
$$-x_1 - x_2 \leq -4$$

Choose entering and leaving variables according to the "largest coefficient rule," i.e. the entering variable is that with the largest coefficient, with ties (in entering or leaving) broken by taking the variable with the smallest subscript.

[11 points] 2. Consider the LP (linear program)

$$\text{maximize} \quad 7x_1 + 5x_2 + 2x_3, \quad \text{subject to } x_1, x_2, x_3 \geq 0 \text{ and}$$
$$x_1 + x_2 + x_3 \leq 5$$
$$x_1 + 3x_2 + x_3 \leq 10$$
$$3x_1 + x_2 + x_3 \leq 6$$

Use complementary slackness to see if $x^* = (1, 3, 0)$ is an optimal solution to the above LP.

[10 points] 3. Consider the LP

$$\text{maximize} \quad 2x_1 + x_2, \quad \text{subject to } x_1, x_2 \geq 0 \text{ and}$$
$$x_1 + x_2 \leq 2$$
$$x_1 - 3x_2 \leq 2$$

Use the perturbation method discussed in class on this LP to find its optimal solution; introduce powers of epsilon into the LP in the first dictionary. Use the largest coefficient rule, as in problem 1. Make sure you begin by adding ϵ to the first dictionary equation and ϵ^2 to the second (not vice versa; don't interchange the inequalities!).

[7 points] 4. The company "Le chocolat délicieux" has you solve an LP, the objective being measured in dollars of profit. You solve the LP, arriving at a final dictionary that is non-degenerate. Then they tell you that their constraint $(.7)x_1 + (.76)x_2 \leq 3000$, based on the 3000 kilos of cocoa ingredients they have, might change because someone offered to sell them a tiny bit of such ingredients at \$5 per kilo. Everything else in the LP will remain the same. How do you determine whether they should buy a tiny bit of such ingredients?