Marks

[8] **1.** Each of the dictionaries below has x_1 entering variable. Below each dictionary, write which variable leaves (if any); also write "unbounded pivot" (meaning that you know the LP is unbounded at that pivot) and/or "degenerate pivot" if these terms apply. DO NOT ACTUALLY PIVOT.

$$\begin{aligned} x_3 &= 8 - x_1 - 2x_2 \\ x_4 &= 5 - x_1 - x_2 \\ z &= 4x_1 - 5x_2 \end{aligned}$$

$$\begin{aligned} x_3 &= 8 - x_1 - 2x_2 \\ x_4 &= 5 + x_1 - x_2 \\ z &= 4x_1 - 5x_2 \end{aligned}$$

$$\begin{aligned} x_3 &= 8 + x_1 - 2x_2 \\ x_4 &= 5 - x_1 - x_2 \\ z &= 4x_1 - 5x_2 \end{aligned}$$

$$\begin{aligned} x_3 &= 8 + x_1 - 2x_2 \\ x_4 &= 5 + x_1 - x_2 \\ z &= 4x_1 - 5x_2 \end{aligned}$$

$$\begin{aligned} x_3 &= 0 + x_1 - 2x_2 \\ x_4 &= 5 - x_1 - x_2 \\ z &= 4x_1 - 5x_2 \end{aligned}$$

$$\begin{aligned} x_3 &= 0 - x_1 - 2x_2 \\ x_4 &= 5 + x_1 - x_2 \\ z &= 4x_1 - 5x_2 \end{aligned}$$

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[12] **2.** Consider the two matrix games

$$A_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$$

For each matrix find (1) the value of the game "Alice Announces," (2) the value of the game "Betty Announces," (3) the duality gap in the "Announce" games, (4) the value and equilibrium strategies of the "Scream" games.

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[6] **3.** Consider the problem: maximize x_1 subject to $x_1 \leq 5$, $x_1 \geq 2$, $x_1 \geq 0$. Write this as a linear program in standard form. Use the two-phase method, **adding an auxilliary variable x_0 to EVERY slack variable equation in the dictionary,** to solve this LP. Use the smallest subscript rule to break any ties for entering or leaving variables.

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[8] **4.** Consider our usual LP: maximize $4x_1 + 5x_2$ subject to $x_1 + 2x_2 \leq 8$, $x_1 + x_2 \leq 5$, $2x_1 + x_2 \leq 8$, and $x_1, x_2 \geq 0$. Write the slack variables for this linear program, and write down the dual linear program and dual slack variables.

Check to see if the following are optimal solutions to the primal linear program using complementary slackness:

(a) $x_1 = 3$, $x_2 = 2$;

(b) $x_1 = 2$, $x_2 = 3$;

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[4] **5.** Say you took one ϵ (i.e., $\epsilon = \epsilon_1 = \epsilon_2 = \cdots$) instead of m different ϵ's (namely $\epsilon_1 >> \epsilon_2 >> \ldots$) in the perturbation method. Explain, using a formula on the note sheet involving A_B^{-1}, how you could get a degenerate pivot.

[6] **6.** Consider the game where Alice and Betty each write down a number from 1 to n (where n is a fixed, positive integer), and then reveal it to the other, with the rule that 1 beats 2, 2 beats 3, 3 beats 4, and so on, until $n-1$ beats n, and finally 1 beats n; if their number are not consecutive or n and 1, then the game is a draw. (Here the winner receives one dollar from the loser, and nothing happens in a draw.)

(a) Explain the relationship of Rock/Paper/Scissors to this game when $n = 3$; be specific about how strategies of one game correspond to those of the other.

(b) Show (as simply as possible) that Alice and Betty playing all strategies $1/n$ of the time give an equilibrium of this game. Be precise about what fact(s) you are using.

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The End

Be sure that this examination has 8 pages including this cover

The University of British Columbia

Midterm Examinations - March 2009

Mathematics 340–202

Closed book examination | Time: 60 minutes

Name _____ **Signature** _____

Student Number _____ **Instructor's Name** _____

Section Number _____

Special Instructions:

THIS EXAM IS TWO-SIDED! You will be given a note sheet. Calculators, other notes, or other aids may not be used. Answer questions on the exam.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		8
2		12
3		6
4		8
5		4
6		6
Total		44