

Marks

- [8] 1. Consider the two matrix games

$$A_1 = \begin{bmatrix} 1 & -1 \\ -4 & 4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & -1 \\ 4 & 4 \end{bmatrix}$$

Assume that A_1 is irreducible (i.e., that every strategy is essential) and use linear algebra to find the value of the game and the equilibrium strategies. How do you know that the irreducibility assumption on A_1 was correct (explain carefully)? Assume that A_2 is irreducible, and try to do the same for A_2 ; how do you know that the irreducibility assumption on A_2 was wrong (explain carefully)?

- [5] 2. Consider the problem: maximize x_1 subject to $x_1 + x_2 \leq 5$, $x_1 \geq 6$, $x_1, x_2 \geq 0$. Write this as a linear program in standard form. Use the two-phase method, **adding an auxiliary variable x_0 to EVERY slack variable equation in the dictionary**, to show that this linear program is infeasible.

- [6] **3.** Consider the problem: maximize $x_1 + x_2$ subject to $x_1 + 2x_2 \leq 4$, $2x_1 + x_2 \leq 5$, and $x_1, x_2 \geq 0$. Write the slack variables for this linear program, and write down the dual linear program and dual slack variables.

Check to see if the following are optimal solutions to the primal linear program using complementary slackness:

(a) $x_1 = 2, x_2 = 1$;

(b) $x_1 = 0, x_2 = 2$;

- [2] 4. Explain why the following linear program must involve a degenerate pivot: maximize x_1 subject to $x_1 \leq x_2 + x_3$, $x_1 + x_2 + 4x_3 \leq 2$, $x_2 + 5x_3 \leq 10$, $3x_1 + 3x_2 + 5x_3 \leq 7$, $x_1, x_2, x_3 \geq 0$.

- [6] 5. Show that for all $m \times n$ matrix games, A , and stochastic \mathbf{s}, \mathbf{t} of dimension m we have

$$\text{Scream}_{\text{Alice}}((\mathbf{s} + \mathbf{t})/2) \geq (1/2)\text{Scream}_{\text{Alice}}(\mathbf{s}) + (1/2)\text{Scream}_{\text{Alice}}(\mathbf{t})$$

Evaluate each term in this formula for the game Rock–Paper–Scissors, where \mathbf{s} represents “play Rock always” and \mathbf{t} represents “play Scissors always.” Does equality hold? [This is standard Rock-Paper-Scissors: rock beats scissors, scissors beats paper, paper beats rock, and each win pays one unit to the winner.]

The End

Be sure that this examination has 7 pages including this cover

The University of British Columbia

Midterm Examinations - March 2008

Mathematics 340–201

Closed book examination

Time: 50 minutes

Name _____ Signature _____

Student Number _____ Instructor's Name _____

Section Number _____

Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

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(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

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2		5
3		6
4		2
5		6
Total		27