

Midterm

[40 points total]

Throughout this entire exam, whenever there is ambiguity in the entering or leaving variable, you are to use the “highest coefficient rule;” this means that you choose the entering variable according to the highest coefficient, with ties (in entering or leaving) broken by taking the variable with the smallest subscript.

[10 points] 1. Use the two-phase method to solve

$$\begin{array}{ll} \text{maximize} & 2x_1 + 3x_2 \\ \text{subject to} & x_1 + x_2 \leq 10 \\ & -x_1 - x_2 \leq -5 \\ & -2x_1 - x_2 \leq -15 \\ & x_1, x_2 \geq 0 \end{array}$$

[10 points] 2. Consider the LP (linear program)

$$\begin{array}{ll} \text{maximize} & x_1 + 2x_2 + 3x_3 \\ \text{subject to} & x_1 + x_2 + x_3 \leq 3 \\ & x_2 + x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Use complementary slackness to see which (if any) of $(x_1^*, x_2^*, x_3^*) =$

$$(0, 0, 3), \quad (1, 0, 2), \quad \text{and/or} \quad (1, 2, 0)$$

is an optimal solution to the above LP.

[10 points] 3. Consider the following dictionaries

$$\begin{array}{ll} \text{(A)} & \text{(B)} \\ x_1 = 2 - x_3 + x_4 & x_1 = 2 - x_3 + x_4 \\ x_2 = 3 + x_3 + 2x_4 & x_2 = 3 + x_3 + 2x_4 \\ x_5 = 4 - 2x_3 + 3x_4 & x_5 = 4 - 2x_3 + 3x_4 \\ z = 10 - x_3 + 2x_4 & z = 10 + x_3 - 2x_4, \end{array}$$

and the dictionaries obtained during the perturbation method

$$\begin{array}{ll} \text{(C)} & \text{(D)} \\ x_3 = 2 + \epsilon - 2x_1 - x_2 & x_1 = \epsilon^2 - \epsilon - x_2 + 2x_3 \\ x_4 = 1 + \epsilon^2 - x_1 + 2x_2 & x_4 = \epsilon^2 - x_2 + 3x_3 \\ z = 2x_1 + x_2 & z = 7 + \epsilon + x_2 - 4x_3 \end{array}$$

- (a) Explain what it means for a dictionary to be feasible. Which of the above four dictionaries are feasible?
- (b) For each feasible dictionary above, perform one iteration of the simplex method. State if the simplex method terminates at the end of or during the iteration, and if so state your conclusion in terms of the z -value and any associated optimal solution.

[10 points] 4. A cappuccino is made from one shot of espresso, three ounces of milk, and six ounces of foam. A latté is made from one shot of espresso, seven ounces of milk, and two ounces of foam. A café sells only cappuccinos and lattés, and makes one dollar profit on each drink it sells. Today the café has materials to produce 50 shots of espresso, 20 ounces of milk, and 30 ounces of foam.

- (a) Write down a linear program to maximize the profit the café will make. Write down the dual LP. For all variables involved (objective, decision, and slack, both in the primal and dual), state in what units they are given.
- (b) Give an economic interpretation to complementary slackness relating espresso shots leftover (a primal slack variable) and the corresponding dual (decision) variable.
- (c) Give an economic interpretation to complementary slackness relating the number of cappuccinos made (a primal decision variable) and the corresponding dual (slack) variable.