Midterm Solutions Math 340/101 Fall 1997

1.

$$\begin{aligned} x_3 &= -2 + x_2 + x_1 + x_0 \\ x_4 &= -1 \qquad\;\; + x_1 + x_0 \\ w &= \qquad\qquad\qquad\quad - x_0 \end{aligned}$$

x_0 enters, x_3 leaves

$$\begin{aligned} x_0 &= 2 - x_2 - x_1 + x_3 \\ x_4 &= 1 - x_2 \qquad + x_3 \\ w &= -2 + x_2 + x_1 - x_3 \end{aligned}$$

x_1 enters, x_0 leaves

$$\begin{aligned} x_1 &= 2 - x_2 - x_0 + x_3 \\ x_4 &= 1 - x_2 \qquad + x_3 \\ w &= \qquad\quad - x_0 \end{aligned}$$

enter phase 2

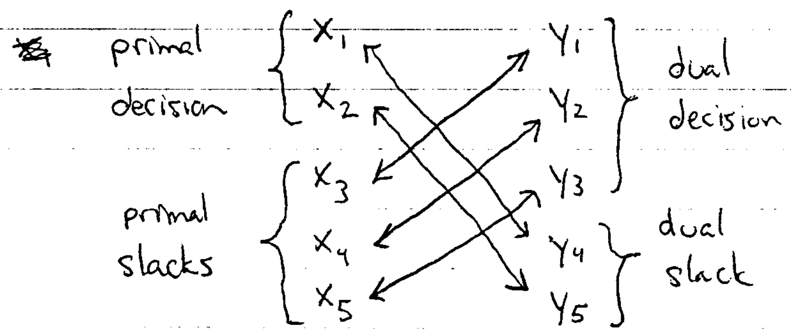
$$\begin{aligned} x_1 &= 2 - x_2 + x_3 \\ x_4 &= 1 - x_2 + x_3 \\ z &= (x_1 + 2x_2) \\ &= 2 + x_2 + x_3 \end{aligned}$$

x_2 enters, x_4 leaves

$$\begin{aligned} x_1 &= 1 + x_4 \\ x_2 &= 1 - x_4 + x_3 \\ z &= 3 - x_4 + 2x_3 \end{aligned}$$

x_3 enters, but nothing leaves. Hence this LP is unbounded.

2. We have a correspondence



Midterm Solutions:

2. (Cont'd) $x^* = (2,3)$ or $(x_1^*, \dots, x_5^*) = (2,3,0,0,1)$, so x_1^*, x_2^*, x_5^* are positive, so corresponding y_4^*, y_5^*, y_3^* are zero, so

$$y_1^* + y_2^* + 2y_3^* = 4 \qquad (\mathrm{I})$$

$$y_1^* + 2y_2^* + y_3^* = 5 \qquad (\mathrm{II})$$

$$y_3^* = 0 \qquad (\mathrm{III})$$

so $y_3^* = 0$, $y_2^* = 1$ (subtracting I from II), and $y_1^* = 3$.
So $(y_1^*, \dots, y_5^*) = (3,1,0,0,0)$ and $(x_1^*, \dots, x_5^*) = (2,3,0,0,1)$ are entirely non-negative and satisfy complementary slackness; hence $x^* = (2,3)$ is optimal.

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3. $x_3 = \varepsilon - x_1 - 3x_2$
$x_4 = \varepsilon^2 - x_1 - x_2$
$z = \quad x_1 + 2x_2$

x_2 enters. Since $\varepsilon^2/1 < \varepsilon/3$ we have x_4 leaves

$$x_3 = (\varepsilon - 3\varepsilon^2) + 2x_1 + 3x_4$$
$$x_2 = \varepsilon^2 - x_1 - x_4$$
$$z = 2\varepsilon^2 - x_1 - 2x_4$$

The optimal solution is therefore $x_1^* = 0$, $x_2^* = \varepsilon^2$, with optimal objective $z^* = 2\varepsilon^2$; taking $\varepsilon \to 0$ gives the optimal to the original LP, $x^* = (0,0)$, $z^* = 0$.