Math 340/102 Midterm Fall 2001 Brief Solutions

1. $x_3 = 10 - x_1 - x_2 + x_0$

$x_4 = -5 + x_1 + x_2 + x_0$

$x_5 = -15 + 2x_1 + x_2 + x_0$

$w = -x_0$

pivot to feasibility

x_0 enters, x_5 leaves

$x_0 = 15 - 2x_1 - x_2 + x_5$

$x_4 = 10 - x_1 + x_5$

$x_3 = 25 - 3x_1 - 2x_2 + x_5$

$w = -15 + 2x_1 + x_2 - x_5$

x_1 enters (highest coefficient)

x_0 leaves

$$x_1 = \frac{15}{2} - \frac{x_0}{2} - \frac{x_2}{2} + \frac{x_5}{2}$$

$$x_4 = \frac{5}{2} + \frac{x_0}{2} + \frac{x_2}{2} + \frac{x_5}{2}$$

$$x_3 = \frac{5}{2} + \frac{3x_0}{2} - \frac{x_2}{2} - \frac{x_5}{2}$$

x_0 retired

$x_1 = 15/2 - x_2/2 + x_5/2$

$x_4 = 5/2 + x_2/2 + x_5/2$

$x_3 = 5/2 - x_2/2 - x_5/2$

$z = 2x_1 + 3x_2$

$= 15 + 2x_2 + x_5$

x_2 enters x_3 leaves

$x_2 = 5 - 2x_3 - x_5$

$z = 25 - 4x_3 - x_5$

$x_1 = 5 + \dots$

$x_4 = \dots$

So $(x_1^*, x_2^*) = (5,5)$ is optimal solution, with $z^* = 25$.

2. Dual is

min $3y_1 + 2y_2$

s.t. $y_1 \geq 1$ (or $-y_1 \leq -1$)

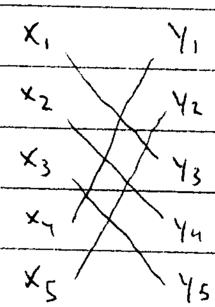
$y_1 + y_2 \geq 2$ etc.

$y_1 + y_2 \geq 3$

Slack variables $y_3 = -1 + y_1$

$y_4 = -2 + y_1 + y_2$

$y_5 = -3 + y_1 + y_2$



$$\left(\begin{array}{l} x_4 = 3 - x_1 - x_2 - x_3 \\ x_5 = 2 - x_2 - x_3 \end{array}\right)$$

Proposed solution: $(x_1^*, x_2^*, x_3^*) = (0,0,3)$

$x_4^* = 0$, $x_5^* = -1$, so $x_5^* < 0$ means $(0,0,3)$ is infeasible.

Proposed solution: $(1,0,2)$. $x_4^* = 0$, $x_5^* = 0$.

So $x_1^*, x_3^* > 0 \Rightarrow y_3^* = y_5^* = 0 \Rightarrow$

$-1 + y_1^* = 0$ and $-3 + y_1^* + y_2^* = 0$

$\Rightarrow y_1^* = 1$, $y_2^* = 2$. Then

$(y_3^* = 0)$ $y_4^* = 1$, $(y_5^* = 0)$ so all x_i^* and $y_j^* \geq 0$ so $(1,0,2)$ is optimal.

2.(Cont') Proposed solution: $(1,2,0)$

Again $x_4^* = x_5^* = 0$. So $x_1^*, x_2^* > 0$
$\Rightarrow y_3^* = 0$ and $y_4^* = 0 \Rightarrow -1 + y_1^* = 0$,
$-2 + y_1^* + y_2^* = 0$ so $y_1^* = y_2^* = 1$.
But $y_5^* = -3 + y_1^* + y_2^* = -1$, so
$(1,2,0)$ not optimal.

3. Feasible (A),(B),(C). Infeasible (D) since $\varepsilon^2 - \varepsilon < 0$. Feasible means all basic variables' constants are ≥ 0.

(A): x_4 enters, nothing leaves, so LP is unbounded

(B): x_3 enters x_1 leaves (ties broken with smallest subscript)

$$x_3 = 2 - x_1 + x_4$$
$$x_2 = 5 - x_1 + 3x_4$$
$$x_5 = 2x_1 + x_4$$
$$z = 12 - x_1 - x_4$$

So dictionary is optimal, $z^* = 12$, $(x_1^*, \ldots, x_5^*) = (0,5,2,0,0)$.

(C) x_1 enters (highest coefficient) x_4 leaves ($\frac{1+\varepsilon^2}{1} < \frac{2+\varepsilon}{2}$), So

$$x_1 = 1 + \varepsilon^2 - x_4 + 2x_2$$
$$x_3 = \varepsilon - 2\varepsilon^2 + 2x_4 - 5x_2$$
$$z = 2 + 2\varepsilon^2 - 2x_4 + 5x_2$$

Simplex method is not finished.

4.(a) x_1 = cappuccinos made, x_2 = ~~espressos~~ lattés made

$$\max z = x_1 + x_2$$
s.t. $x_1 + x_2 \leq 50$ (espresso bound)
$3x_1 + 7x_2 \leq 20$ (milk bound)
$6x_1 + 2x_2 \leq 30$ (foam bound)

So x_1 = cappuscinos, x_2 = lattes
x_3 = espresso, x_4 = milk, x_5 = foam, z = \$.

Dual: $\min w = 50y_1 + 20y_2 + 30y_3$
s.t. $y_1 + 3y_2 + 6y_3 \geq 1$ (\$/capp.)
$y_1 + 7y_2 + 2y_3 \geq 1$ (\$/latté)

So w = \$, y_1 = \$/espresso, y_2 = \$/(ounce)milk
y_3 = \$/ounce foam, y_4 = \$/capp., y_5 = \$/latté

(b) If there are leftover espresso shots (i.e. $x_3^* > 0$) then the value of a marginal espresso shot (at optimum) (i.e. y_1^*) is zero.

(c) If, at optimal, cappucinos are being made (i.e. $x_1^* > 0$), then the value of its components (one espr. shot, 3 oz. milk, 6 oz. foam) equals the profit per cappucino (i.e. $y_4^* = 0$).

$\boxed{\text{OR}}$ If the value of the components exceeds the profit for a cappucino, we can't be making any cappucino.