Midterm 2000 Fall, M340/102 Brief solutions

1. $x_3 = 1 - x_1 + x_2 + x_0$

$x_4 = -4 + 2x_1 - 3x_2 + x_0$

x_0 enters, x_4 leaves

$x_0 = 4 - 2x_1 + 3x_2 + x_4$

$x_3 = 5 - 3x_1 + 4x_2 + x_4$

$w = -x_0 = -4 + 2x_1 - 3x_2 - x_4$

x_1 enters, x_3 leaves

$x_1 = 5/3 - 1/3\, x_3 + 4/3\, x_2 + 1/3\, x_4$

$x_0 = 2/3 + 2/3\, x_3 + 1/3\, x_2 + 1/3\, x_4$

$w = -2/3 - 2/3\, x_3 - 1/3\, x_2 - 1/3\, x_4$

So original LP infeasible.

2. $x_3 = 3 - x_1 - \dots$

$x_4 = -4 + 2x_1 - \dots$ } The rest is same as in #1

$\leftarrow$ same

x_0 = same

$x_3 = 7 - 3x_1 + \dots$ rest same

x_1 enters, x_0 leaves

$x_1 = 2 - \frac{1}{2} x_0 + \frac{3}{2} x_2 + \frac{1}{2} x_4$

$x_3 = 1 + \frac{3}{2} x_0 - 1/2\, x_2 - \frac{1}{2} x_4$

$z = x_1 - 2x_2$

$= 2 - \frac{1}{2} x_2 + \frac{1}{2} x_4$

x_4 enters, x_3 leaves

$x_4 = 2 - x_2 - 2x_3$

$x_1 = 3$ + irrelevant

$z = 3 - x_2 - x_3$

Opt. sol. is $(3,0)$, $z = 3$

3. $x_1 = 2$ $\quad y_1$

$x_2 = 0$ $\quad y_2 = 0$

$x_3 = 2$ $\quad y_3$

$x_4 = 0$ $\quad y_4 = 0$

$x_5 = 6$ $\quad y_5$

$x_6 = 0$ $\quad y_6 = 0$

$$\left.\begin{aligned} y_1 + 4y_3 &= 1 \\ 2y_1 + 2y_3 &= 1 \end{aligned}\right\} \quad y_1 = \frac{1}{3} \quad y_3 = \frac{1}{6}$$

$$y_5 = -1 + y_1 + 6y_2 + 2y_3$$

$$= -1 + \frac{1}{3} + 0 + \frac{1}{3} = \frac{-1}{3}$$

So $y_5 < 0$ so proposed solution has no dual feasible solution to match. Hence $(2,0,2)$ not an optimal solution.

4. x_2 enters; since $\varepsilon - \varepsilon^2 > \varepsilon^2$, x_4 leaves

$$x_2 = \varepsilon^2 - x_4 + 3x_3$$
$$x_1 = \varepsilon - 2\varepsilon^2 + x_4 - x_3$$
$$z = 7 + \varepsilon + \varepsilon^2 - x_4 - x_3$$

z value increases from $7+\varepsilon$ to $7+\varepsilon+\varepsilon^2$, a change of ε^2.
If $\varepsilon = 0$, the change would be 0, a degenerate iteration.

5. A degenerate dictionary is one where at least one basic variable is zero (in its corresponding feasible solution). If a dictionary is not degenerate, then the entering variable always increases (from 0 to a positive value), thereby increasing z = the objective. Cycling requires the z value to stay the same, requiring all dictionaries along the cycle to be degenerate.