Marks

[15] **1.** Use the two-phase method to solve

$$\text{maximize} \quad -2x_1 - x_2 \quad \text{subject to} \quad x_1, x_2 \geq 0,$$

$$\begin{array}{rrl} x_1 & -x_2 & \leq -1, \\ -x_1 & -x_2 & \leq -3. \end{array}$$

Use the largest coefficient rule to choose the entering variable, breaking ties by choosing the smallest subscript. Clearly state which variable enters and which leaves on each pivot, and what your final dictionary means in terms of the original problem.

[15] **2.** Consider the problem

$$\text{maximize} \quad 3x_1 + x_2 + x_3 \quad \text{subject to} \quad x_1, x_2, x_3, x_4 \geq 0,$$

$$\begin{array}{rrrrl} x_1 & +x_2 & +x_3 & +x_4 & \leq 6, \\ x_1 & -x_2 & & +2x_4 & \leq 4, \\ x_1 & & +x_3 & & \leq 2, \end{array}$$

Confirm that $x_1^* = 2$, $x_2^* = 4$, $x_3^* = 0$ and $x_4^* = 0$ is an optimal solution to the above LP using complementary slackness. Carefully state every condition that you are verifying; for each equation you write down for a hypothetical dual optimal solution, carefully indicate where it comes from.

[20] **3.** Consider the problem

$$\text{maximize} \quad 3x_1 + 2x_2 + 4x_3 \quad \text{subject to} \quad x_1, x_2, x_3 \geq 0,$$

$$\begin{array}{rrrl} x_1 & +x_2 & +2x_3 & \leq 4, \\ 2x_1 & & +3x_3 & \leq 5, \\ 2x_1 & +x_2 & +3x_3 & \leq 7. \end{array}$$

Solve this problem using the revised simplex method as described in class (in particular, do not compute B^{-1} explicitly; instead, use the eta factorization). Use the smallest subscript rule (not the largest coefficient rule!) to select your entering and leaving variables.

[30] **4.** Consider the problem

$$\text{maximize} \quad c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \quad \text{subject to} \quad x_1, x_2, x_3, x_4 \geq 0,$$

$$\begin{array}{rrrrl} x_1 & +2x_2 & +3x_3 & +x_4 & \leq b_1, \\ x_1 & +x_2 & +2x_3 & +3x_4 & \leq b_2, \end{array}$$

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where

$$c_1 = 5, \quad c_2 = 6, \quad c_3 = 9, \quad c_4 = 8, \quad b_1 = 5, \quad b_2 = 3.$$

This optimal dictionary and B^{-1} (as in the revised simplex method) for this problem is

$$\begin{array}{llllll} x_1 & = 1 & -x_3 & -5x_4 & +x_5 & -2x_6, \\ x_2 & = 2 & -x_3 & +2x_4 & -x_5 & +x_6, \\ z & = 17 & -2x_3 & -5x_4 & -x_5 & -4x_6, \end{array} \qquad B^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

[5](a) If we let c_1 be arbitrary (all other c's and b's fixed as above), for what range of values of c_1 is this dictionary (adjusted for the change in c_1) still optimal?

[5](b) If we let b_2 be arbitrary (all other c's and b's fixed as above), for what range of values of b_2 is this dictionary (adjusted for the change in b_2) still optimal?

[5](c) Let us take the original problem with $b_2 = 7$. Form a new (not necessarily feasible) dictionary based on incorporating this change into the old optimal dictionary. Perform one pivot to reach a new optimal dictionary. What is the new optimal solution?

[5](d) Now we add the constraint $3x_1 + 4x_2 + 6x_3 + 4x_4 \le 10$ to the original problem. Form a new (not necessarily feasible) dictionary based on adding this constraint to the old optimal dictionary. Perform one pivot to reach a new optimal dictionary. What is the new optimal solution?

[10](e) Now $b_2 = t$ becomes a parameter. Solve the resulting LP for all real values of t, and make a plot of $z^*(t)$.

[20] **5.** Consider the problem

$$\text{maximize } 2x_1 + x_2 \qquad \text{s.t. } x_1 + x_2 \le 3, \qquad x_1, x_2 \ge 0.$$

Write down primal and dual dictionaries for this problem, with the dual variables relabelled so that the i-th primal variable corresponds to the i-th relabelled dual variable under complementary slackness. Use the Lemke-Howson algorithm to solve this problem. Clearly state which variable enters and which variable leaves every dictionary, and **state why they have been chosen to enter and leave**.

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Formulae

The following formulae may be of use. They will not be explained; you are assumed to understand what they mean and to what they refer.

$$\begin{array}{rll} x_B = & B^{-1}b & -B^{-1}A_N x_N \\ z = & c_B B^{-1} b & +(c_N - c_B B^{-1} A_N) x_N \end{array}$$

$$y = c_B B^{-1}, \qquad yB = c_B, \qquad d = B^{-1}a, \qquad Bd = a, \qquad x_B^* - td$$

The End

Be sure that this examination has 3 pages including this cover

The University of British Columbia

Final Examinations - Dec., 1997

Mathematics 340–101

Closed book examination

Time: $2\frac{1}{2}$ hours

Name ______________________

Signature ______________________

Student Number ______________________

Instructor's Name ______________________

Section Number ______________________

Special Instructions:

Candidates may not use any notes or calculators. A list of formulae is provided at the end of this exam. Answer questions in the booklets provided.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

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Total		100