

- [5] **2.** Give an example of a dictionary for which the perturbation method must make a different pivot depending on whether $\epsilon_1 \ll \epsilon_2$ or $\epsilon_2 \ll \epsilon_1$. Explain why your dictionary will pivot differently in the two situations. (You do not have to give the linear program; just write down one dictionary.)

- [5] **3.** The two phase method and the dual simplex method (with dual pivots) are two ways to deal with dictionaries with negative constants. When would you choose one over the other? Explain.

- [5] 4. How can you modify a matrix game, A , to get a new matrix game with all positive entries and the same equilibrium strategies? How do you know the equilibria are the same? Why did we find this useful for the simplex method?

- [5] 5. Imagine that the revised simplex method, for some i we have a factorization of $A_{B_i}^{-1}$ (i.e., some ability to efficiently multiply it by a vector). How can we perform calculations involving $A_{B_{i+1}}^{-1}$, $A_{B_{i+2}}^{-1}$, etc., using this factorization and eta matrices? In practice, will we use this factorization and eta matrices from this point on, or will we refactor from scratch every so often? Why?

- [5] **6.** You are given data $(x_1, y_1), (x_2, y_2), \dots, (x_{20}, y_{20})$, and wish to find the a, b such that $\max_i |a + bx_i - y_i|$ is as small as possible. Write a linear program that will find the desired a, b . For how many i do you expect that the maximum $|a + bx_i - y_i|$ will be achieved? Explain.

- [5] **7.** Consider the dictionary in the parameter, t :

$$\begin{array}{rcllclcl} x_1 & = & 2t - 5 & -x_3 & -5x_4 & +x_5 & -2x_6 \\ x_2 & = & 5 - t & -x_3 & +2x_4 & -x_5 & +x_6 \\ z & = & 4t + 5 & -2x_3 & -5x_4 & -x_5 & -4x_6 \end{array}$$

For t slightly greater than 5, what is the new optimal z value? Find this using (part or all of) one dual pivot.

[8] 8. Imagine “Arnold sings,” “Bob sings,” “Arnold dances,” “Bob dances,” have respective benefits to society of 7, 10, 16, 22. We wish to match Arnold and Bob with the activities “sings” and “dances” for the maximum total benefit.

(a) Write a linear program that expresses this problem.

(b) Write down the dual linear program, and interpret it as trying to “price” Arnold and Bob so that each activity gets a “best deal.”

(c) Explain how complementary slackness tells you when a pricing scheme gives you an optimal solution.

- [10] 9. Consider the linear program: maximize $x_1 + 2x_2$ subject to $x_1 \leq 3$, $x_2 \leq 4$, $x_1 + x_2 \leq 6$, and $x_1, x_2 \geq 0$. Use complimentary slackness to check if the following solutions are optimal.
- (a) $x_1 = 3$, $x_2 = 3$.

(b) $x_1 = 2$, $x_2 = 4$.

- [10] **10.** For every real α find the value and equilibria strategies for the matrix game:

$$\begin{bmatrix} 3 & 2 \\ \alpha & 4 \end{bmatrix}.$$

Graph the value of the game as a function of α . Find the duality gap for the announce (pure strategy) game as a function of α .

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The University of British Columbia

Final Examinations - April 2010

Mathematics 340–201

Closed book examination

Time: 150 minutes

Name _____ Signature _____

Student Number _____ Instructor's Name _____

Section Number _____

Special Instructions:

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1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

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3. Smoking is not permitted during examinations.

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