Marks

[8] **1.** Use the two-phase method to solve:

$$\begin{aligned} \text{maximize} \quad & x_1 + x_2, \quad \text{subject to } x_1, x_2 \geq 0 \text{ and} \\ & x_1 + 2x_2 \leq 10 \\ & -x_1 - 2x_2 \leq -4 \end{aligned}$$

When there is more than one possible entering or leaving variable, choose the one with the smallest subscript.

$x_3 = 10 - x_1 - 2x_2 + x_0$

$x_4 = -4 + x_1 + 2x_2 + x_0$

x_0 enters, x_4 leaves

$x_0 = 4 - x_1 - 2x_2 + x_4$

$x_3 = 14 - 2x_1 - 4x_2 + x_4$

x_1 enters, x_0 leaves

$x_1 = 4 - x_0 - 2x_2 + x_4$

$x_3 = 6 + 2x_0 + 0x_2 - x_4$

x_0 disappears

$x_1 = 4 - 2x_2 + x_4$

$x_3 = 6 - x_4$

$z = 4 + x_4$

x_4 enters, x_3 leaves

$x_4 = 6 - x_3$

$x_1 = 10 - 2x_2 - x_3$

$z = 10 - x_3$

done! opt sol: $x_1 = 10$, $x_2 = 0$, $z = 10$

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[2] **2.** Consider the dictionary:

$$x_3 = 3 + 4x_4 - 5x_1$$
$$x_2 = 9 + 7x_4 + 5x_1$$
$$z = 30 + 5x_4 - x_1$$

Find a setting of x_1, x_2, x_3, x_4 that is feasible and that makes $z = 5030$.

Take $x_4 = 1000$, $x_1 = 0$; $x_2 = 7009$, $x_3 = 4003$

[8] **3.** Give a short explanation as you answer the following questions.

(a) In a zero sum game, which strategies are pareto optimal?

If any player strictly gains, the other player strictly loses, so no strategy is pareto dominated and all strategies are pareto optimal.

(b) When the simplex method cycles, do you necessarily encounter degenerate pivots?

Yes — the objective never decreases in the simplex method, so cycling $\Rightarrow$ objective stays the same, i.e. have degenerate pivots.

(c) In a matrix game with probability vector, γ, δ, what is the relationship hold between the two quantities:

$$\text{Scream}_A((\gamma+\delta)/2) \quad \text{and} \quad (\text{Scream}_A(\gamma) + \text{Scream}_A(\delta))/2.$$

$\min_{\text{comp}}(\gamma^T M + \delta^T M)/2 \geq \min_{\text{comp}}(\gamma^T M)/2 + \min_{\text{comp}}(\delta^T M)/2$

So $\text{Scream}_A((\gamma+\delta)/2) \geq \text{Scream}_A(\gamma)/2 + \text{Scream}_A(\delta)/2$

(d) Give an example of an application of part (c) to the game battleship done in class.

Let γ be any player A (square choosing) strategy, and δ its top $\leftrightarrow$ bottom exchange. Then $\text{Scream}_A(\gamma) = \text{Scream}_A(\delta)$, so screaming $(\gamma+\delta)/2$ is at least as good as screaming γ. So Max Scream_A is attained at a strategy that looks the same top $\leftrightarrow$ bottom.

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[8] **4.** Consider the matrix game associated to the matrix

$$M = \begin{bmatrix} 1 & 2 \\ 3 & c \end{bmatrix},$$

where c is a given real number.

(a) Assuming that all pure strategies are involved in a unique equilibrium, what is player A's equilibrium strategy, $\vec{\alpha} = (\alpha_1, \alpha_2)$?

(b) For what values of c is it not the case that all pure strategies are involved in an equilibrium? Use domination to explain what are the equilibria for those values of c.

(a) $(\alpha_1, \alpha_2)\begin{bmatrix} 1 & 2 \\ 3 & c \end{bmatrix} = [v \; v]$ and $\alpha_1 + \alpha_2 = 1$, so also

$\alpha_1 + 3\alpha_2 = v = 2\alpha_1 + c\alpha_2$ so $\alpha_1 + (c-3)\alpha_2 = 0$. So

$(1-\alpha_2) + (c-3)\alpha_2 = 0$ so $\alpha_2 = \frac{1}{4-c}$; $\alpha_1 = \frac{3-c}{4-c}$.

(b) If $c \geq 2$, bottom row dominates top, and B plays col 1 if $3 \leq c$ or col 2 if $3 \geq c$. So equilibria are

$c \geq 3$: $\vec{\alpha} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\vec{\beta} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$2 \leq c \leq 3$: $\vec{\alpha} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\vec{\beta} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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[6] **5.** Consider the movie game between Arnold and Maria, with payout matrix:

	Maria to Bambi	Maria to Terminator
Arnold to Bambi	$(2, 1)$	$(0, 0)$
Arnold to Terminator	$(0, 0)$	$(1, 2)$

(where (a, b) means Arnold gets utility a, Maria utility b). This can be alternatively written:

	Maria to Terminator	Maria to Bambi
Arnold to Bambi	$(0, 0)$	$(2, 1)$
Arnold to Terminator	$(1, 2)$	$(0, 0)$

(a) Describe a sense in which the above movie game is symmetric between Arnold and Maria.

Arnold $\leftrightarrow$ Maria and Bambi $\leftrightarrow$ Terminator gives same game

(b) Given a game:

	Maria cooks pasta	Maria cooks rice
Arnold cooks quiche	(a_{11}, b_{11})	(a_{12}, b_{12})
Arnold cooks ratatouille	(a_{21}, b_{21})	(a_{22}, b_{22})

write down what it should mean for the game to be symmetric (i.e., write down the equations), such that the above movie game and any zero sum symmetric game are symmetric in your sense.

$a_{ij} = b_{ji}$ for all i, j, since then Arnold's payout when he plays i and Maria j is same as Maria's payout when she plays i and he j.

(c) Are all Nash equilibria in a symmetric game necessarily symmetric? Either show this is the case, or else give a counterexample. [Hint: you might think of the homework problem on this game.]

No: we have seen Arnold $\to$ Bambi 100% and Maria $\to$ Bambi 100% is an equilibrium which, when Arnold exchanged with Maria and Bambi with Terminator, is not the same equilibrium (it is one of the other three on the homework; Arnold $\to$ Terminator 100% and Maria $\to$ Terminator 100%).

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The End

Be sure that this examination has 6 pages including this cover

The University of British Columbia

Midterm Examinations - March 2007

Mathematics 340–202

Closed book examination | Time: 50 minutes

Name _____ **Signature** _____

Student Number _____ **Instructor's Name** _____

Section Number _____

Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

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2		2
3		8
4		8
5		6
Total		32