M340/101 Fall 1997

Midterm

[40%] 1. Use the two-phase method to solve

$$\begin{aligned} \text{maximize} \quad & x_1 + 2x_2, \quad \text{subject to } x_1, x_2 \geq 0 \text{ and} \\ & -x_1 - x_2 \leq -2 \\ & -x_1 \leq -1 \end{aligned}$$

Choose entering and leaving variables according to Anstee's rule, i.e. the largest coefficient rule with ties broken by taking the variable with the smallest subscript.

[30%] 2. Consider our distinguished LP

$$\begin{aligned} \text{maximize} \quad & 4x_1 + 5x_2, \quad \text{subject to } x_1, x_2 \geq 0 \text{ and} \\ & x_1 + x_2 \leq 5 \\ & x_1 + 2x_2 \leq 8 \\ & 2x_1 + x_2 \leq 8 \end{aligned}$$

An expert on beverage makers tells you that she believes $x^* = (2, 3)$ is an optimal solution. Use complementary slackness to find the predicted dual optimal solution and to demonstrate that $x^* = (2, 3)$ is indeed optimal.

[30%] 3. Consider the LP

$$\begin{aligned} \text{maximize} \quad & x_1 + 2x_2, \quad \text{subject to } x_1, x_2 \geq 0 \text{ and} \\ & x_1 + 3x_2 \leq 0 \\ & x_1 + x_2 \leq 0 \end{aligned}$$

Illustrate the perturbation method discussed in class on this LP; i.e. solve this LP with the simplex method, using the perturbation method and Anstee's rule to choose the entering and leaving variables. Make sure you begin adding ϵ to the first dictionary equation and ϵ^2 to the second (not vice versa; don't interchange the inequalities!).