

Midterm

[40 points total]

[8 points] 1. Use the two-phase method to solve

$$\begin{array}{ll} \text{maximize} & x_1 - 2x_2 \\ \text{subject to} & x_1 - x_2 \leq 1 \\ & -2x_1 + 3x_2 \leq -4 \\ & x_1, x_2 \geq 0 \end{array}$$

Choose entering and leaving variables according to the “largest coefficient rule,” i.e. the entering variable is that with the largest coefficient, with ties (in entering or leaving) broken by taking the variable with the smallest subscript.

[8 points] 2. Use the two-phase method to solve

$$\begin{array}{ll} \text{maximize} & x_1 - 2x_2 \\ \text{subject to} & x_1 - x_2 \leq 3 \\ & -2x_1 + 3x_2 \leq -4 \\ & x_1, x_2 \geq 0 \end{array}$$

using the same “largest coefficient rule” as in problem 1.

[10 points] 3. Consider the LP (linear program)

$$\begin{array}{ll} \text{maximize} & x_1 + x_2 + x_3 \\ \text{subject to} & x_1 + x_2 + 2x_3 \leq 6 \\ & 3x_1 + 6x_2 + 3x_3 \leq 18 \\ & 4x_1 + 2x_2 + 2x_3 \leq 12 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Use complementary slackness to see if $(x_1, x_2, x_3) = (2, 0, 2)$ is an optimal solution to the above LP.

[8 points] 4. Consider the dictionary obtained while applying the perturbation method,

$$\begin{array}{l} x_1 = \epsilon - \epsilon^2 - x_2 + 2x_3 \\ x_4 = \epsilon^2 - x_2 + 3x_3 \\ z = 7 + \epsilon + x_2 - 4x_3 \end{array}$$

Perform one iteration of the simplex method, obtaining a new dictionary. What is the change in the z value from the above dictionary to the new dictionary? Would your iteration be degenerate if ϵ were set to 0, i.e. if all ϵ 's were not present?

[6 points] 5. What is meant by a “degenerate dictionary”? Explain why it is impossible for the simplex method to cycle without encountering degenerate dictionaries.