Math 340/102 Midterm Brief Solutions — Fall 1999

1. $x_3 = 6 - 3x_1 - 2x_2$
$x_4 = -4 + x_1 + x_2$
$z = x_1 + 2x_2$

infeasible, so add x_0's

$x_3 = 6 - 3x_1 - 2x_2 + x_0$
$x_4 = -4 + x_1 + x_2 + x_0$
$w = -x_0$

x_0 enters, x_4 leaves

$x_0 = 4 - x_1 - x_2 + x_4$
$x_3 = 10 - 4x_1 - 3x_2 + x_4$
$w = -4 + x_1 + x_2 - x_4$

x_1 enters, x_3 leaves

$x_1 = \frac{5}{2} - \frac{1}{4}x_3 - \frac{3}{4}x_2 + \frac{1}{4}x_4$
$x_0 = \frac{3}{2} + \frac{1}{4}x_3 - \frac{1}{4}x_2 + \frac{3}{4}x_4$
$w =$ minus last line

x_2 enters, x_1 leaves

$x_2 = \frac{10}{3} - \frac{1}{3}x_3 - \frac{4}{3}x_1 + \frac{1}{3}x_4$
$x_0 = \frac{2}{3} + \frac{1}{3}x_3 + \frac{1}{3}x_2 + \frac{2}{3}x_4$
$w = -\frac{2}{3} - \frac{1}{3}x_3 - \frac{1}{3}x_2 - \frac{2}{3}x_4$

So optimal w is $-3/2$ so original problem is infeasible

Check: $\frac{1}{3}$(x_3 const) $+ \frac{2}{3}$(x_4 constraint) is

$$\begin{array}{rl} \frac{1}{3}(3x_1 + 2x_2 \le 6) & \\ + \frac{2}{3}(-x_1 - x_2 \le -4) & \\ \hline \frac{1}{3}x_1 \quad \le -\frac{2}{3} & \text{which is infeasible} \end{array}$$

2.

$x_1^* = 1$	y_1^* must $= 0$
$x_2^* = 3$	y_2^*
$x_3^* = 0$	y_3^*
$x_4^* = 1$	y_4^* must $= 0$
$x_5^* = 0$	y_5^* must $= 0$
$x_6^* = 0$	y_6^*

So original solution is feasible, and for dual:

$0 = y_4^* = -7 + y_1^* + y_2^* + 3y_3^*$
$0 = y_5^* = -5 + y_1^* + 3y_2^* + y_3^*$

and $y_1^* = 0$ so

$y_2^* + 3y_3^* = 7$
$3y_2^* + y_3^* = 5$ so $y_2^* = 1, y_3^* = 2$

Check all y^*'s are ≥ 0:

$y_1^* = 0,\ y_2^* = 1,\ y_3^* = 2,\ y_4^* = y_5^* = 0,$

$y_6^* = -2 + y_1^* + y_2^* + y_3^* = 1$ so OK.

ALTERNATIVELY: Check dual feasibility from inequalities:

$y_1 + y_2 + 3y_3 \ge 7$
$y_1 + 3y_2 + y_3 \ge 5$
$y_1 + y_2 + y_3 \ge 2$

checks out with $(y_1, y_2, y_3) = (0, 1, 2)$ so OK.

Since derived dual solution is feasible, original solution is optimal.

3. $x_3 = 2+\varepsilon - x_1 - x_2$

$x_4 = 2+\varepsilon^2 - x_1 + 3x_2$

$z = \quad 2x_1 + x_2$

x_1 enters, x_4 leaves

$x_1 = 2+\varepsilon^2 - x_4 + 3x_2$

$x_3 = \varepsilon - \varepsilon^2 + x_4 - 4x_2$

$z = 4 + 2\varepsilon^2 - 2x_4 + 7x_2$

x_2 enters, x_3 leaves

$x_2 = \frac{\varepsilon-\varepsilon^2}{4} + \frac{1}{4}x_4 - \frac{1}{4}x_3$

$x_1 = 2+\varepsilon^2 + \frac{3}{4}(\varepsilon-\varepsilon^2) +$ whatever

$z = 4 + 2\varepsilon^2 + \frac{7}{4}(\varepsilon-\varepsilon^2) - \frac{1}{4}x_4 - \frac{7}{4}x_3$

So optimal is $(x_1^*, x_2^*) = \left(2+\varepsilon^2+\frac{3}{4}(\varepsilon-\varepsilon^2), \frac{\varepsilon-\varepsilon^2}{4}\right)$

with $z^* = 4 + \frac{7}{4}\varepsilon + \frac{1}{4}\varepsilon^2$. Letting $\varepsilon \to 0$ gives

$(x_1^*, x_2^*) = (2, 0)$, $z^* = 4$.

4. The constraint corresponds to a slack variable, say x_i.

If x_i is basic in final dictionary, there is a surplus of cocoa ingredients and such ingredients have no marginal value. So we won't buy.
If x_i is non-basic in final dictionary, look at its coefficient in the objective (i.e. z) row. If the coefficient is >5 we buy, since its marginal value to us is $>\$5$ per kilo. If it is <5 we don't buy.
If it $= 5$ we don't care.