MATH 340: STUDY GUIDE AND SAMPLE PROBLEMS FOR THE FINAL EXAM

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Contents

All homework problems should be considered as Sample Exam Problems, as well as all Sample Exam Problems listed in the two course documents (one on Matrix Games, the other on Eta Matrices). The following document contains supplemental Sample Exam Problems.

1. What We Covered in Math 340

Broadly speaking, here is an outline of the topics covered in Math 340. Note that these topics have some overlap. For example, to know that the perturbation method works, you need to know that if B is a set of basic variables in any dictionary of the simplex method, then A_B is invertible.

(1) Simplex Method Algorithms, such as
   (a) set up any LP as an LP in standard form;
   (b) the simplex method for an LP (linear program) in standard form that is feasible;
   (c) the perturbation method (to guarantee that the simplex method won't cycle);
   (d) the two-phase method (for an LP in standard form that isn't feasible).
(2) Duality Theory
   (a) How to form and interpret the dual of a general LP.
   (b) Complementary slackness between primal and dual variables.
   (c) Its interpretation in game theory
(3) Matrix Notation for the Simplex Method; principles of the revised simplex method.
(4) Applications

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   (a) Game theory (we spent a lot of time on this).
   (b) Eta matrices and aspects of the revised simplex method.
   (c) Weighted Bipartite Matching, i.e., The Assignment Problem (also 3-dimensional matching).
   (d) Regression.
   (e) Other applications: matrix games whose rows are concave up functions of the row number (where the best mixed strategy involves either or both of the top and bottom rows) or concave down functions of the row number (where the best mixed strategy is at the top row, or bottom row, or two adjacent rows).
(5) Linear Programming without Linear Programming, meaning that you reason about an LP without actually running the simplex method, typically by doing the following:
   (a) you describe a particular application as a linear program;
   (b) you possibly eliminate some variables or constraints that are dominated by others (this is analogous in game theory to eliminating rows or columns that are dominated by others); and
   (c) you reach some general conclusion about the optimal solution based on some broad principles, namely that
      (i) the number of variables in the basis never changes, and
      (ii) if A_B is the set of columns in "big A" corresponding to the collection of basic variables, B, in any dictionary of the simplex method, then A_B must be invertible.

2. Learning Goals and Sample Final Problems:

Here is what you should know and what you should be able to do for the final exam. This outline should match up with the outline of the previous section; it will be expanded a bit.

If you look at past exams, you will see that the basic algorithms and principles tend to reappear frequently; however, each year the specific topics beyond the basics vary a lot.

This year things differ as follows: (1) revised simplex was explained in broad terms, (2) many more applications were discussed.

(1) Simplex Method:
   (a) set up any LP as an LP in standard form;
   (b) the simplex method for a feasible LP in standard form;
      (i) apply the algorithm [Mid 97(1) (which is unbouded),Mid 01(3)(A,B), Mid 09(1), Fin 09(2a)];
      (ii) explain how it works;
      (iii) explain that when you reach a final dictionary you know the answer to your LP and you can prove that this is the case [Mid 07(2), Fin 99(1), Fin 99(2b), Fin 00(1b,1c)].
   (c) the perturbation method and degenerate pivots:

(i) apply the algorithm [Mid 97(3), Mid 99(3), Mid 01(3)(C,D)];
(ii) explain why it works (to avoid cycling and degenerate pivots) [Mid 00(4), Mid 00(5), Mid 07(3b), Mid 09(5), Fin 09(6)];
(iii) explain when degenerate pivots must occur [Mid 08(4)].

(d) the two-phase method [Mid 99(1), which is infeasible, Mid 00(1), which is infeasible, Mid 00(2), which is feasible, Mid 01(1), Mid 07(1) feasible and bounded, Fin 97(1), Fin 99(2a), Fin 00(1a,1c), Mid 08(2) infeasible, Fin 08(4), Mid 09(3), Fin 09(4)].
(i) apply the method;
(ii) explain why it works;

(2) Duality Theory
(a) How to form and interpret the dual of a general LP [See complementary slackness exam questions].
(i) Given a guess of an optimal solution, $\mathbf{x}^*$, for the primal and an optimal solution, $\mathbf{y}^*$, for the dual, check if these are both correct by checking if the guesses are feasible and give objectives that are equal. [This is very easy, much easier than the next item, which takes only one guess.] [Mid 14(2b).]
(b) Complementary slackness between primal and dual variables:
(i) Use complementary slackness to verify if a proposed optimal solution really is optimal [Mid 97(2), Mid 99(2), Mid 00(3), Mid 01(2), Fin 97 (2), Mid 08(3), Fin 08(3), Mid 09(4)].
(ii) Its interpretation in game theory [Fin 09(7a,b)].

(3) Matrix Notation for the Simplex Method. This involves representing an LP in standard form as $\max \vec{c} \cdot \vec{x}$ subject to $A\vec{x} \leq \vec{b}$ and $\vec{x} \geq \vec{0}$, and the equations

$$A_B x_B + A_N x_N = \vec{b}$$

to represent a general dictionary. The revised simplex method is covered in Chapter 8 of Vanderbei and the handout on eta matrices [Problems 3.4 and 3.16.] For other Sample Exam Problems on matrix notation, see its applications to the perturbation method and to "Linear Programming Without Linear Programming."

(4) Applications
(a) Game theory (we spent a lot of time on this).
(i) Solve 2×2 matrix games [Mid 07(4), Mid 08(1), Fin 08(1), Mid 09(2), Fin 09(3)].
(b) Eta matrices and aspects of the revised simplex method [Fin 09(1a), Fin 09(2b,c)]. We also observed that the dictionary description of the simplex method can also be given by row operations on the associated "tableaux."

   (c) Weighted Bipartite Matching, i.e., The Assignment Problem (also 3-dimensional matching). [Problems 3.5, 3.11–3.13 of the Handout on Eta Matrices.]
   (d) Regression. [Problems 3.7, 3.8 of the Handout on Eta Matrices.]
   (e) Other applications: matrix games whose rows are concave up functions of the row number (where the best mixed strategy involves either or both of the top and bottom rows) or concave down functions of the row number (where the best mixed strategy is at the top row, or bottom row, or two adjacent rows). [Problems 3.14, 3.15.]
(5) Linear Programming without Linear Programming, meaning that
   (a) you describe a particular application as a linear program;
   (b) you possibly eliminate some variables or constraints that are dominated by others (this is analogous in game theory to eliminating rows or columns that are dominated by others); and
   (c) you reach some general conclusion about the optimal solution based on some broad principles, namely that
      (i) the number of variables in the basis never changes, and
      (ii) if A_B is the set of columns in "big A" corresponding to the collection of basic variables, B, in any dictionary of the simplex method, then A_B must be invertible.

   [Problems 3.5–3.11 of the Handout on Eta Matrices, Homework 5 (1)]

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