Math 340–101 Fall 2014

**Homework #5**

1. Consider the matrix game,

$$A = \begin{bmatrix} 1 & 16 \\ 4 & 9 \\ 9 & 4 \\ 16 & 1 \end{bmatrix},$$

and consider the task of finding Alice's optimal strategy, $[p_1\ p_2\ p_3\ p_4]$.

(a) Argue that the value, $v$, of this game is a bounded, positive number.

(b) Write down a linear program to determine this value, $v$, and the optimum strategy $[p_1\ p_2\ p_3\ p_4]$, and argue that the condition

$$p_1 + p_2 + p_3 + p_4 = 1$$

can be replaced with the condition

$$p_1 + p_2 + p_3 + p_4 \leq 1$$

(why?).

(c) Argue that Alice has an optimal strategy with at most two of the $p_i$'s being non-zero.

(d) Find the value and optimal mixed strategies of the matrix game consisting of the first and last row of the above matrix game, i.e., the game

$$A' = \begin{bmatrix} 1 & 16 \\ 16 & 1 \end{bmatrix}$$

in any way you like.

(e) Using the optimal mixed strategies in in $A'$, what happens if Betty plays the same mixed strategy in the game $A$, and Alice plays the same strategy, meaning that Alice doesn't play the second or third row? Do you get optimal mixed strategies in $A$?

(f) If $A$ is a $100 \times 2$ matrix game, and your intuition tells you that Alice's best mixed strategy uses only rows 12 and 29, describe a relatively quick method to see if your intuition is correct.

2. Final Exam, April 2009, Problem 4.