Math 340–101 Fall 2014

Homework #4

1. Use the two-phase method to find the solution of the following LP problem:

$$\text{maximize } \zeta = 3x_1 + x_2,$$
$$\text{subject to} \quad \begin{array}{rrcl} x_1 & +x_2 & \leq & 1 \\ -2x_1 & +x_2 & \geq & 1 \\ 3x_1 & +2x_2 & \geq & 4 \end{array}$$
$$\text{and} \qquad x_1, x_2 \geq 0.$$

Choose entering and leaving variables by taking the variable with the smallest subscript amoung all viable candidates; this is often called the "smallest subscript rule" or "Bland's rule." Hint: you should find that the LP is infeasible after roughly three pivots.

2. Same problem as problem (1) for the LP:

$$\text{maximize } \zeta = 3x_1,$$
$$\text{subject to} \quad \begin{array}{rcl} x_1 & \leq & 7 \\ x_1 & \geq & 1 \\ x_1 & \geq & 4 \end{array}$$
$$\text{and} \qquad x_1 \geq 0.$$

After you have solved this correctly ($x_1 = 7$, $z = 21$), go back to the first pivot of the first phase, where x_0 enters the basis, and make an incorrect choice of leaving variable; what is wrong with the resulting dictionary?

3. (a) Apply the perturbation method to the LP:

$$\text{maximize} \quad 2x_1 + x_2, \quad \text{subject to}$$
$$x_1 \leq 2$$
$$x_2 \leq 3$$
$$x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

taking x_2 to enter the basis on your first pivot; specifically add ϵ_1 to the first inequality (writing $x_1 \leq 2+\epsilon_1$), ϵ_2 to the second, and ϵ_3 to the third, with "$1 >> \epsilon_1 >> \epsilon_2 >> \epsilon_3$. Find the maximum and draw a picture of feasible region, and indicate what your simplex steps look like in the picture.

(b) Do the same thing with ϵ_1 added to the third inequality and ϵ_3 added to the first (e.g., writing $x_1 \leq 2 + \epsilon_3$).

(c) How does the simplex method differ from part (a) to part (b)? Does it make sense that two perturbations of the above LP can give different dictionaries, even when we ignore the ϵ's in the dictionaries?