Math 340–101 Fall 2014

Homework #3

1. Consider a matrix game, A. Let $\mathbf{x}$ be a stochastic vector such that each entry of $\mathbf{x}^{\mathrm{T}}A$ is at least v_1. Similarly, let $\mathbf{y}$ be a stochastic vector such that each entry of $A\mathbf{y}$ is at most v_2.

   (a) Argue that the value of "Alice announces a mixed strategy" is at least v_1.

   (b) Similarly argue that the value of "Betty announces a mixed strategy" is at most v_2.

   (c) Argue that one can never have $v_1 > v_2$.

   (d) Argue that if $v_1 = v_2$, then the value of the game (i.e., of the mixed strategy games) is v_1.

   (e) Consider the following matrix game (called "even/odd pennies" in the handout):

   $$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}.$$

   Let's say that you guess that Alice's best mixed strategy is [2/3 1/3] and that Betty's best mixed strategy is [1/2 1/2]. Use Items (a)—(d) above to argue that the value of this game is 0.

   (f) Say that for a matrix game, A, you believe that $\mathbf{x}$ is an optimum mixed strategy for Alice, and that $\mathbf{y}$ is an optimum mixed strategy for Betty. Describe a quick calculation to test whether or not your belief is really true.

2. Consider the LP from class:

   $$\max 4x_1 + 5x_2, \quad \text{s.t. } x_1 + x_2 \le 5, \quad x_1 + 2x_2 \le 8,$$

   $$2x_1 + x_2 \le 8, \quad x_1, x_2 \ge 0.$$

   Solve this using the simplex method (with dictionaries, as in class), but this time let x_1 enter in the first dictionary to obtain the second.

3. Consider the following even/odd penny game Recall the even/odd penny game, where Alice and Betty each hold either one or two pennies:

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix},$$

and consider the matrix, A_{+10}, you get by adding 10 to each entry, and the matrix, A_{-10} by subtracting the same:

$$A_{+10} = \begin{bmatrix} 11 & 9 \\ 8 & 12 \end{bmatrix}, A_{-10} = \begin{bmatrix} -9 & -11 \\ -12 & -8 \end{bmatrix},$$

(a) Solve the linear program: maximize v subject to $v, x_1, x_2 \geq 0$ and

$$v \leq 11x_1 + 8x_2, \quad v \leq 9x_1 + 12x_2, \quad x_1 + x_2 \leq 1$$

using the simplex method.

(b) Solve the linear program: maximize v subject to $v, x_1, x_2 \geq 0$ and

$$v \leq -9x_1 - 12x_2, \quad v \leq -11x_1 - 8x_2, \quad x_1 + x_2 \leq 1$$

using the simplex method. [Hint: This should not take long.]

(c) In the LP of part (a), argue that it does not matter if we impose $v \geq 0$ or place no restriction on v; argue that it does not matter if we write $x_1 + x_2 \leq 1$ or $x_1 + x_2 = 1$.

(d) Explain why the linear program in part (a) gives Alice's optimal mixed strategy for the game A_{+10}, in view of part (c) and discussion in class or the notes.

(e) Explain how Alice's and Betty's optimal mixed strategies and the value of the game are related between the games A, A_{+10}, and A_{-10} (briefly justify your explanation).

(f) Why doesn't the LP in part (b) give Alice's optimal mixed and the game value for A_{-10}? In other words, what works for A_{10} that breaks down for A_{-10}?

(g) Recall Betty plays $[1/2 \;\; 1/2]^{\mathrm{T}}$ in optimal mixed. Can you find this strategy "hidden" somewhere in your final dictionary for part (a)? Can you explain this?