HOMEWORK #5 (SAMPLE MIDTERM QUESTIONS), MATH 223, SPRING 2019

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All previous homework problems are sample mideterm questions. In addition the following type of questions may appear on the midterm.

Homework Problems

Please do Problem 1 below, and at least one of Problems 2 or 3

(1) (a) Let

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 5 \\ 4 \end{bmatrix}, \quad \boldsymbol{\delta}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{\delta}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(b) Consider the fact that $\mathbf{v}_1 = 3\boldsymbol{\delta}_1 + 3\boldsymbol{\delta}_2$: write down an analogous formula for $\mathbf{v}_2$. You don't need to explain anything; just write down a formula.

(c) Organize the above two equations to get a set of equations that looks like

$$\mathbf{v}_1 = c_1\boldsymbol{\delta}_1 + c_2\boldsymbol{\delta}_2$$
$$\mathbf{v}_2 = c_3\boldsymbol{\delta}_1 + c_4\boldsymbol{\delta}_2$$

where $c_1, c_2, c_3, c_4 \in \mathbb{R}$.

(d) Given that $\mathbf{v}_1 = 3\boldsymbol{\delta}_1 + 3\boldsymbol{\delta}_2$, solve for $\boldsymbol{\delta}_1$ in terms of $\mathbf{v}_1$ and $\boldsymbol{\delta}_2$.

(e) Using your answers to the above two parts, write a formula for $\mathbf{v}_2$ in term of $\mathbf{v}_1$ and $\boldsymbol{\delta}_2$.

(f) Combine your answers to the above to write a set of equations that looks like

$$\boldsymbol{\delta}_1 = c_1\mathbf{v}_1 + c_2\boldsymbol{\delta}_2$$
$$\mathbf{v}_2 = c_3\mathbf{v}_1 + c_4\boldsymbol{\delta}_2$$

(g) With a similar type of calculation, derive a table that looks like

$$\boldsymbol{\delta}_1 = c_1\mathbf{v}_1 + c_2\mathbf{v}_2$$
$$\boldsymbol{\delta}_2 = c_3\mathbf{v}_1 + c_4\mathbf{v}_2$$

(h) Write

$$1984\boldsymbol{\delta}_1 + 2019\boldsymbol{\delta}_2$$

in terms of $\mathbf{v}_1$ and $\mathbf{v}_2$.

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(i) Is there a connection between your answer to the previous part and the solution to the 2×2 system of equations

$$\begin{aligned} 3x_1 + 5x_2 &= 1984 \\ 3x_1 + 4x_2 &= 2019 \end{aligned}$$

(j) Check your solution by hand or computer.

(2) Find a solution to the 2×2 system

$$\begin{aligned} 3x_1 + 5x_2 &= b_1 \\ 3x_1 + 4x_2 &= b_2 \end{aligned}$$

(for general $b_1, b_2 \in \mathbb{R}$) using the method of basis exchange as in Problem 1.

(3) Write down explicit solutions (using TEX or LATEXor by hand) to the following 2020 2×2 systems of equations:

$$\begin{aligned} 3x_1 + 5x_2 &= 1984 \\ 3x_1 + 4x_2 &= 0 \end{aligned}$$

and

$$\begin{aligned} 3x_1 + 5x_2 &= 1984 \\ 3x_1 + 4x_2 &= 1 \end{aligned}$$

and

$$\begin{aligned} 3x_1 + 5x_2 &= 1984 \\ 3x_1 + 4x_2 &= 2 \end{aligned}$$

and ... and

$$\begin{aligned} 3x_1 + 5x_2 &= 1984 \\ 3x_1 + 4x_2 &= 2019 \end{aligned}$$

using the method of basis exchange as in Problem 1.

For additional problems, create your own 2×2 systems, 7×7 systems, etc.

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