HOMEWORK #3, MATH 223, SPRING 2019

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Homework Problems

(1) Exercises 2,4,8,9 from Section 2.4 (Test) of the textbook.

(2) Let V, W be real vector spaces; we say that a map $\mathcal{L}\colon V \to W$ is a *linear transformation* if for all $v_1, v_2 \in V$ and $\alpha, \beta \in \mathbb{R}$ we have that
$$\mathcal{L}(\alpha v_1 + \beta v_2) = \alpha\mathcal{L}(v_1) + \beta\mathcal{L}(v_2).$$
Prove that in this case
$$\ker(\mathcal{L}) \stackrel{\mathrm{def}}{=} \{v \in V \mid \mathcal{L}(v) = 0\}$$
is a subspace of V. Prove that
$$\mathrm{Image}(\mathcal{L}) \stackrel{\mathrm{def}}{=} \{\mathcal{L}(v) \mid v \in V\}$$
is a subspace of W.

(3) Let $\mathcal{L}\colon V \to V$ be the linear transformation on Functions$(\mathbb{Z} \to \mathbb{R})$ given by
$$(\mathcal{L}f)(n) = f(n+2) - 2f(n+1) + f(n).$$
Show that the functions (1) $f(n) = 1$ for all $n \in \mathbb{Z}$, and (2) $f(n) = n$ for all $n \in \mathbb{Z}$ are in the kernel of $\mathcal{L}$. Show that given $f(0), f(1)$ one can give an exact formula for the function $f \in \mathrm{Ker}(\mathcal{L})$ whose those values at $0, 1$.

(4) Generalize the problem above to
$$(\mathcal{L}f)(n) = f(n+3) - 3f(n+2) + 3f(n+1) - f(n):$$
Show that (1) any polynomial of degree at most 2 is in the kernel of $\mathcal{L}$, and (2) given $f(0), f(1), f(2)$, one ca give an exact formula for the function $f \in \ker(\mathcal{L})$ with those values at $0, 1, 2$.

(5) Let $V = \mathcal{P}_3 = \mathrm{Poly}_{\le 3}(\mathbb{R})$. Which of the following subsets of V are subspaces? Explain.
   (a) $\{p \in V \mid p(3) - 4p(5) = 0\}$
   (b) $\{p \in V \mid p(3) - 4p(5) = 3\}$
   (c) $\{p \in V \mid p(3)p(4) - 4p(5) = 0\}$
   (d) $\{p \in V \mid p(3)p(4) - 4p(5)p(6) = 0\}$
   (e) $\{p \in V \mid p'(3) = 0\}$
   (f) $\{p \in V \mid p'(3) + p(4) = 0\}$

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