WRITTEN HOMEWORK 5 (SOLUTIONS), MATH 200, FALL 2015

ALBERT CHAU, JOEL FRIEDMAN, BEN KRAUSE, AND DANG KHOA NGUYEN

This homework **may be modified from section to section!** Check your section's website for any modifications to this homework for your section.

Final 2013WT2, Problem 2(a)

At $t = 0$ we have $(x, y, z) = (0, -1, 1)$,

$$P = \frac{0^2 + 2(-1)^2}{1 + 1^2} = 1, \quad T = 5 + 0 \cdot (-1) - 1^2 = 4.$$

We have

$$\frac{d}{dt}(PT)^2 = 2(PT)(\frac{dP}{dt}T + P\frac{dT}{dt}) = 2(1 \cdot 4)(4\frac{dP}{dt} + 1\frac{dT}{dt}) = 32\frac{dP}{dt} + 8\frac{dT}{dt}.$$

As for derivatives, we have $dx/dt = 2$, $dy/dt = 2t$, $dz/dt = -\sin t$, so at $t = 0$ we have $dx/dt = 2$, $dy/dt = 0$, $dz/dt = 0$. So

$$\frac{dP}{dt} = P_x\frac{dx}{dt} + P_y\frac{dy}{dt} + P_z\frac{dz}{dt},$$

which at $t = 0$ is

$$= 2P_x + 0P_y + 0P_z.$$

So we only need to compute P_x (at $t = 0$), which is

$$\frac{\partial}{\partial x}\frac{x^2 + 2y^2}{1 + z^2} = \frac{2x}{1 + z^2},$$

which at $t = 0$ is therefore $(2 \cdot 0)/(1 + 1^1) = 0$. Hence at $t = 0$ we have

$$\frac{dP}{dt} = 2 \cdot 0 = 0.$$

Similarly, at $t = 0$ we have

$$\frac{dT}{dt} = 2T_x + 0T_y + 0T_z.$$

So we only need to compute T_x (at $t = 0$), which is

$$\frac{\partial}{\partial x}(5 + xy - z^2) = y$$

which at $t = 0$ is therefore -1. Hence at $t = 0$ we have

$$\frac{dT}{dt} = 2(-1) = -2.$$

It follows that at $t = 0$ we have

$$\frac{d}{dt}(PT)^2 = 32\frac{dP}{dt} + 8\frac{dT}{dt} = 32 \cdot 0 + 8 \cdot (-2) = -16.$$

Remark: if you don't notice that $dy/dt = 0$ and $dz/dt = 0$ at $t = 0$, then you might think you need to compute P_y and P_z to find

$$\frac{dP}{dt} = P_x\frac{dx}{dt} + P_y\frac{dy}{dt} + P_z\frac{dz}{dt};$$

this is why you might want to first compute dx/dt, dy/dt, and dz/dt, and see if you get lucky.

Final 2013WT1, Problem 1(b)

(i).
For $F(x, y, z) = x^2z^3 + y\sin(\pi x) + y^2$ we have

$$F_x = 2xz^3 + y\pi\cos(\pi x), \quad F_y = \sin(\pi x) + 2y, \quad F_z = x^2 3z^2.$$

Therefore at $(1, 1, -1)$ we have

$$F_x = 2(1)(-1)^3 + 1\pi(-1) = -2 - \pi, \quad F_y = \sin(\pi) + 2 = 2, \quad F_z = 3.$$

Therefore the tangent plane to $F(x, y, z) = 0$ at $(1, 1, -1)$ is given by

$$F_x(1, 1, -1)(x - 1) + F_y(1, 1, -1)(y - 1) + F_z(1, 1, -1)(z + 1) = 0,$$

that is,

$$(-2 - \pi)(x - 1) + 2(y - 1) + 3(z + 1) = 0.$$

(ii).
Since $F(x, y, z) = 0$ with F as above, we can differentiate implicitly to find

$$F_x + F_z z_x = 0,$$

and hence at $(1, 1, -1)$, from part (i), we have

$$z_x = -F_x/F_z = (2 + \pi)/3.$$

(iii).
By the linear approximation we have

$$\Delta z \approx (\Delta x)z_x + (\Delta y)z_y = (-0.03)(2 + \pi)/3 + 0z_y = -0.01(2 + \pi).$$

Final 2012WT1, Problem 2

(a).
We have $z = f(x, y) = 1000 - 0.02x^2 - 0.01y^2$, and so $z_x = -0.04x$ and $z_y = -0.02y$. At $x = 0$ and $y = 100$ we have

$$\nabla z = (-0.04 \cdot 0, -0.02 \cdot 100) = (0, -2).$$

Therefore the direction of steepest ascent, i.e., the direction in which ∇z points, is the negative y direction, i.e., South.

(b).

The slope of the hill in the steepest ascent direction is $|\nabla z| = |(0, -2)| = 2$.

(c).

The slope in the steepest descent direction is -2, which means at 5 m/s your rate of change is -10 m/s (which is -36 km/h, so you should be wearing a helmet).

Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, CANADA.

Department of Computer Science, University of British Columbia, Vancouver, BC V6T 1Z4, CANADA, and Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, CANADA.

E-mail address: jf@cs.ubc.ca or jf@math.ubc.ca

URL: http://www.math.ubc.ca/~jf

Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, CANADA.

Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, CANADA.