WRITTEN HOMEWORK 3, MATH 200, FALL 2015

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This homework **may be modified from section to section!** Check your section's website for any modifications to this homework for your section.

Problem 1: Final 2013WT2, Problem 1

There are a number of ways of solving these problems; we shall give one way.

(a). The direction of the line is the cross product of the normals:

$$\langle -2,1,1\rangle \times \langle -1,3,3\rangle = \langle 0,5,-5\rangle.$$

To find a particular point on the line we may set $z=0$ in the equations for W_1 and W_2 (since the z-component of line's direction is non-zero) to find the point $\langle x,y,0\rangle$ where

$$-2x+y=7 \quad \text{and} \quad -x+3y=6,$$

which gives $x=-3$, $y=1$, and hence the equation is:

$$\langle -3,1,0\rangle + t\langle 0,5,-5\rangle$$

(b). Setting

$$\frac{x}{2} = \frac{2y-4}{4} = z+5 = t,$$

we have $x=2t$, $y=2t+2$, and $z=t-5$. Hence a parametric representation of the line is

$$\langle x,y,z\rangle = \langle 0,2,-5\rangle + t\langle 2,2,1\rangle.$$

So M points in the direction $\langle 2,2,1\rangle$ and contains the point $\langle 0,-2,-5\rangle$. Hence both L and M lie on a plane with normal

$$\langle 2,2,1\rangle \times \langle 0,5,-5\rangle = \langle -15,10,10\rangle.$$

Since $\langle -3,1,0\rangle$ lies on L, we have that L lies on the plane

$$-15x+10y+10z = (-15)(-3)+10(1)+(10)(0) = 55\ ;$$

since M contains the point $\langle 0,2,-5\rangle$, M lies on the plane

$$-15x+10y+10z = (-5)(0)+10(2)+(10)(-5) = -30\ ;$$

It follows that the distance between L and M is $|-30-55|/|\langle -15,10,10\rangle| = 17/\sqrt{17} = \sqrt{17}$.

(c). For any values x and y, the point $\langle x, y, z\rangle$ that lies on W_2 is given by

$$z = 2 + (1/3)x - y \ .$$

The region $0 \le x \le 3$ and $0 \le y \le 2$ has

(1) one corner at $x = 0$ and $y = 0$; the z value required to have this point lie on W_2 is $z = 2 + (1/3)(0) - (0) = 2$; hence $\langle 0, 0, 2\rangle$ is one vertex of this parallelogram;
(2) similarly another corner at $x = 3$ and $y = 0$, which gives the parallelogram vertex with $z = 2 + (1/3)3 - (0) = 3$, i.e., the vertex $\langle 3, 0, 3\rangle$; hence one side of the parallelogram is
$$\langle 3, 0, 3\rangle - \langle 0, 0, 2\rangle = \langle 3, 0, 1\rangle \ ;$$
(3) another corner is at $x = 0$, $y = 2$, with $z = 2 + (1/3)(0) - 2 = 0$ at the parallelogram corner, i.e., $\langle 0, 2, 0\rangle$; hence the second side points in the direction
$$\langle 0, 2, 0\rangle - \langle 0, 0, 2\rangle = \langle 0, 2, -2\rangle.$$

Hence the area of the parallelogram is

$$|\langle 3, 0, 1\rangle \times \langle 0, 2, -2\rangle| = |\langle -2, 6, 6\rangle| = \sqrt{76} = 2\sqrt{19}.$$

Problem 2: Final 2013WT1, Problem 1(a)

(i).
We have $x = 2 + 3t$, $y = 4t$, and $z = -1$; solving for t gives

$$\frac{x-2}{3} = \frac{y}{4} \quad \text{and} \quad z = -1$$

(since z is independent of t).

(ii).
L points in the direction $\mathbf{v} = \langle 3, 4, 0\rangle$ (the t coefficients in the parametric form for L). The normal to the plane points in the direction $\mathbf{n} = \langle 1, -1, 2\rangle$ (the coefficients from $x - y + 2z = 0$). Hence the angle between L and the normal to the plane is given by

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{n}}{|\mathbf{v}|\,|\mathbf{n}|} = \frac{-1}{5\cdot\sqrt{6}}.$$

Hence $\alpha = 90° - \theta$ where $\theta = \cos^{-1}(-1/(5\sqrt{6}))$, or $\alpha = \theta - 90°$ for a number between 0 and 90°.

Problem 3: Final 2012WT1, Problem 1

(i). Similar to Problem 1 above, the line L has the direction that is the cross product of the normals to the planes, i.e.,

$$\langle 1, 1, 1\rangle \times \langle 1, -1, 2\rangle = \langle 3, -1, -2\rangle.$$

To find a point on the intersection take $z = 0$ to get the point $\langle x, y\rangle$ such that

$$x + y = 6 \quad \text{and} \quad x - y = 0,$$

which is the point $x = 3$ and $y = 3$ (and $z = 0$). Hence L is the line

$$\langle 3, 3, 0\rangle + t\langle 3, -1, -2\rangle$$

in other words

$$\langle 3 + 3t, 3 - t, -2t\rangle.$$

To find the intersection of L with the coordinate plane $z = 0$ we solve $-2t = 0$, giving $t = 0$, which gives us the point

$$\langle 3 + 3(0), 3 - (0), -2(0)\rangle = \langle 3, 3, 0\rangle.$$

To find the intersection of L with the coordinate plane $y = 0$ we solve $3 - t = 0$, giving $t = 3$, which gives us the point

$$\langle 3 + 3(3), 3 - (3), -2(3)\rangle = \langle 12, 0, -6\rangle.$$

To find the intersection of L with the coordinate plane $x = 0$ we solve $3+3t = 0$, giving $t = -1$, which gives us the point

$$\langle 3 + 3(-1), 3 - (-1), -2(-1)\rangle = \langle 0, 4, 2\rangle.$$

(ii).

A normal to the plane $y = z$, in other words $0x + 1y - 1z = 0$ is the vector $\langle 0, 1, -1\rangle$. Hence the direction of the line we are seeking is perpendicular to this normal vector and the direction of L (which is $\langle 3, -1, -2\rangle$); hence the line we are seeking points in the direction

$$\langle 0, 1, -1\rangle \times \langle 3, -1, -2\rangle = \langle -3, -3, -3\rangle$$

Hence a parametric equation for the line we are seeking is

$$\langle 10, 11, 13\rangle + t\langle -3, -3, -3\rangle.$$

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