

WRITTEN HOMEWORK 2, MATH 200, FALL 2015

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This homework **may be modified from section to section!** Check your section's website for any modifications to this homework for your section.

Problem 1

We now have four ways to determine if two vectors are parallel: the two on Written Homework 1, and the following two methods:

- (1) \mathbf{a} and \mathbf{b} are parallel iff

$$\mathbf{a} \cdot \mathbf{b} = \pm |\mathbf{a}| |\mathbf{b}|,$$

and

- (2) \mathbf{a} and \mathbf{b} are parallel iff

$$\mathbf{a} \times \mathbf{b} = \mathbf{0}$$

Use both of the above two methods to determine

- (1) if $\mathbf{a} = \langle 1, 3, 2 \rangle$ is parallel to $\mathbf{b} = \langle 5, 15, 10 \rangle$;
- (2) if $\mathbf{a} = \langle 1, 3, 2 \rangle$ is parallel to $\mathbf{b} = \langle 5, 15, 12 \rangle$; and
- (3) if $\mathbf{a} = \langle 1, 3, 2 \rangle$ is parallel to $\mathbf{b} = \langle -6, -18, -12 \rangle$.

Problem 2

We shall use the fact (Exercise 53, Section 12.3) that in \mathbb{R}^2 (i.e., the plane), the distance from a point $P_1(x_1, y_1)$ to the line $ax + by + c = 0$ in the (x, y) -plane is

$$(1) \quad \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Say that the equation $ax + by + c = 0$ (as in Problem 2) is *normalized* if $a^2 + b^2 = 1$.

- (1) How does Equation 1 simplify if we know that $a^2 + b^2 = 1$, i.e., if the equation is normalized? Explain.
- (2) The equation of the line in the plane

$$3x + 4y - 2 = 0$$

can be divided by $|\langle 3, 4 \rangle| = 5$ to get an equivalent equation

$$(0.6)x + (0.8)y - (0.4) = 0$$

which is normalized. Using a similar idea, write an equation that is equivalent to

$$5x + 12y - 26 = 0$$

that is normalized. Similarly for the equation

$$8x - 6y + 15 = 0.$$

- (3) Recall the “number of operations” as explained on Problem 4 of [Written Homework 1](#), and in their (soon to be published) solutions. If you are given an equation of one line, $ax + by + c = 0$ and 1000 points whose distance from the line you wish to compute, what is the advantage—in terms of computation speed (i.e., numbers of operations) in first normalizing the equation $ax + by + c = 0$? Explain.

Problem 3

Consider the formula:

$$\text{proj}_{\mathbf{a}} \mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \right) \mathbf{a} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \right) \mathbf{a}$$

- (1) How does this formula simplify if \mathbf{a} happens to be a unit vector?
- (2) Given a single vector, \mathbf{a} , and 1000 vectors whose projection onto \mathbf{a} we wish to compute, can we speed up this computation (in terms of number of operations, as in Problem 2) by first computing $\mathbf{u} = \mathbf{a}/|\mathbf{a}|$? Explain.

Problem 4

Solve Problem 45 part (a) of Section 12.4 of Stewart’s textbook, by drawing a diagram that depicts the various points and vectors (and their lengths, if relevant), and any relevant angles (and their sines or cosines, if relevant). In particular, your diagram should indicate the distance d .

Problem 5

Solve Problem 46 part (a) of Section 12.4 of Stewart’s textbook, by drawing a diagram that depicts the various points and vectors (and their lengths, if relevant), and any relevant angles (and their sines or cosines, if relevant). In particular, your diagram should indicate the distance d .

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