WRITTEN HOMEWORK 1, MATH 200, FALL 2015

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This homework **may be modified from section to section!** Check your section's website for any modifications to this homework for your section.

Recall (Section 12.2) that nonzero vectors two vectors are *parallel* if they are scalar multiples of each other.

Problem 1

One way to determine if two vectors are parallel is to see if one is a scalar multiple of another. For example, to see if $\mathbf{a} = \langle 1, 3, 2\rangle$ is parallel to $\mathbf{b} = \langle 2, 6, 5\rangle$, one writes the equation

$$\mathbf{b} = c\,\mathbf{a}\ ,$$

for a scalar (i.e.,real number) c and sees if there is a solution. In this case the equation amounts to

$$\langle 2, 6, 5\rangle = c\,\langle 1, 3, 2\rangle = \langle c, 3c, 2c\rangle,$$

which means that we are looking for a c such that

$$2 = c, \quad 6 = 3c, \quad 5 = 2c$$

which has no solution. On the other hand if $\mathbf{d} = \langle 10, 30, 20\rangle$, then $\mathbf{d}$ and $\mathbf{a}$ are parallel, since the equation

$$\langle 10, 30, 20 = c\,\langle 1, 3, 2\rangle = \langle c, 3c, 2c\rangle$$

has a solution, namely $c = 10$.

Use the above method to determine

(1) if $\mathbf{a} = \langle 1, 3, 2\rangle$ is parallel to $\mathbf{b} = \langle 5, 15, 10\rangle$;
(2) if $\mathbf{a} = \langle 1, 3, 2\rangle$ is parallel to $\mathbf{b} = \langle 5, 15, 12\rangle$; and
(3) if $\mathbf{a} = \langle 1, 3, 2\rangle$ is parallel to $\mathbf{b} = \langle -6, -18, -12\rangle$.

Problem 2

Recall (Section 12.2) that to each nonzero vector $\mathbf{a}$, the vector $\mathbf{u} = \mathbf{a}/|\mathbf{a}|$ is the unique unit vector (i.e., vector of length 1) whose direction is the same as $\mathbf{a}$.

For each vector below, find the unique unit vector that points in the same direction:

(1) $\mathbf{a} = \langle 3, 0, -4\rangle$;

(2) $\mathbf{b} = \langle 6, 0, 8\rangle$;
(3) $\mathbf{c} = \langle -3, 0, 4\rangle$;
(4) $\mathbf{d} = \langle 3, 0, 4\rangle$;
(5) $\mathbf{e} = \langle 2, 2, 1\rangle$;
(6) $\mathbf{f} = \langle 4, 4, 2\rangle$;
(7) $\mathbf{g} = \langle 2, 6, 3\rangle$; and
(8) $\mathbf{h} = \langle -4, -12, -6\rangle$.

Problem 3

Based on your answer to Problem 2, answer the following questions.

(1) Which vectors in Problem 2 have the same direction?
(2) Which pairs vectors in Problem 2 have the opposite direction? [Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ have the opposite direction if $\mathbf{a}$ and $-\mathbf{b}$ have the same direction.]
(3) Which vectors in Problem 2 are parallel? [To be parallel is equivalent to having the same direction or the opposite direction.]

Problem 4

A computer is given 1000 3-dimensional vectors and must detect which are parallel. Consider the total number of operations—additions, subtractions, multiplications, divisions, and square roots, each counted as one operation—in the following two methods:

(1) (Using the ideas of Problems 2 and 3:) Find the associated unit vectors of the 1000 vectors. [You would still need to compare them, presumably by sorting the unit vectors, but let's ignore this cost.]
(2) Test all pairs of the 1000 vectors for being parallel, by the method of Problem 1. [There are 499,500 pairs of the 1000 vectors.]

Which method takes fewer operations? Would the difference be more significant with 1,000,000 vectors [there would be roughly 5.0×10^{11} pairs of vectors in this case]?

[As an example, the calculation

$$\sqrt{5^2 + (-4)^2} = \sqrt{5 \cdot 5 + (-4) \cdot (-4)}$$

requires one addition (+), two multiplications (two $\cdot$'s) and one square root, for a total of four operations.]

Remark

The idea of determining whether or not vectors are parallel and finding vectors' associated unit vector will be needed in Section 12.5 and exam problems based on Chapter 12.

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