Formula Sheet for Math 200, Section 102, Fall 2015

$$\langle a_1, a_2, a_3\rangle \cdot \langle b_1, b_2, b_3\rangle = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}|\,|\mathbf{b}|\,\cos\theta$$

$$\mathrm{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\right)\mathbf{a}$$

$$\mathrm{Det}(\mathbf{a},\mathbf{b}) = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 = -\begin{vmatrix} b_1 & b_2 \\ a_1 & a_2 \end{vmatrix} = -\mathrm{Det}(\mathbf{b},\mathbf{a})$$

$$\mathrm{Det}(\mathbf{a},\mathbf{b},\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$\mathrm{Det}(\mathbf{a},\mathbf{b},\mathbf{c}) = \mathbf{a}\cdot\left\langle \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix}, -\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix}, \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \right\rangle = \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}).$$

$$\mathbf{b}\times\mathbf{c} = \left\langle \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix}, -\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix}, \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \right\rangle$$

$$|\mathbf{b}\times\mathbf{c}| = |\mathbf{b}|\,|\mathbf{c}|\,|\sin\theta|$$

$$\mathrm{Det}(\mathbf{a},\mathbf{b},\mathbf{c}) = -\mathrm{Det}(\mathbf{b},\mathbf{a},\mathbf{c}) \quad \text{so} \quad \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = -\mathbf{b}\cdot(\mathbf{a}\times\mathbf{c})$$

$$x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct, \qquad \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

$$\mathbf{r}_0 + t\mathbf{v}, \quad (1-t)\mathbf{r}_0 + t\mathbf{r}_1$$

$$\mathbf{n}\cdot(\mathbf{r}-\mathbf{r}_0) = 0, \quad a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$\mathrm{dist}(ax+by+cz+d_1 = 0, ax+by+cz+d_2 = 0) = \frac{|d_1-d_2|}{|\langle a,b,c\rangle|}$$

Distance from a point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is

$$D = |ax_1 + by_1 + cz_1 + d| \,/\, \sqrt{a^2+b^2+c^2}.$$

$$\begin{aligned}
1/2 &= \sin(30^\circ) = \sin(\pi/6) = \cos(60^\circ) = \cos(\pi/3)\\
1/\sqrt{2} &= \sin(45^\circ) = \sin(\pi/4) = \cos(45^\circ) = \cos(\pi/4)\\
\sqrt{3}/2 &= \cos(30^\circ) = \cos(\pi/6) = \sin(60^\circ) = \sin(\pi/3)
\end{aligned}$$

$$\cos(2\alpha) = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\begin{array}{rll}
L(x,y) = f(a,b)+ & f_x(a,b)(x-a)+ & f_y(a,b)(y-b)\\
f(x,y) - f(x_0,y_0) = z - z_0 = & f_x(x_0,y_0)(x-x_0) & +f_y(x_0,y_0)(y-y_0)\\
\Delta f = \Delta z = & f_x(x_0,y_0)\Delta x & +f_y(x_0,y_0)\Delta y\\
df = dz = & f_x(x_0,y_0)\,dx & +f_y(x_0,y_0)\,dy
\end{array}$$

$$\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \cdots + \frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_i}$$

Directional derivative ($\mathbf{u}$ a unit vector):

$$D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u} = \langle f_x, f_y, f_z\rangle \cdot \mathbf{u}$$

$$F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) = 0$$

$$f_{xx}, \quad D = f_{xx}f_{yy} - (f_{xy})^2$$

Local min: $f_{xx} > 0$, $D > 0$; local max: $f_{xx} < 0$, $D > 0$; saddle: $D < 0$; degenerate/indeterminate: $D = 0$.

Lagrange multipliers to maximize/minimize $f = f(x,y)$ or $f = f(x,y,z)$ subject to $g = C =$ Constant where $g = g(x,y)$ or $g = g(x,y,z)$:

$$\nabla f = \lambda \nabla g(, \quad g = C).$$

Mass and centre of mass, density $\rho = \rho(x,y)$:

$$m = \int\int_D \rho(x,y)\,dA, \quad \overline{x} = (1/m)\int\int_D x\rho(x,y)\,dA, \quad \overline{y} = (1/m)\int\int_D y\rho(x,y)\,dA$$

Polar/cylindrical coordinates: $x = r\cos\theta$, $y = r\sin\theta$,

$$dA = dx\ \mathrm{d}y = r\,dr\,d\theta, \quad dV = dx\,dy\,dz = r\,dz\,dr\,d\theta$$

Spherical (like polar in x, y with radius $r = \rho\sin\phi$, $z = \rho\cos\phi$, so ϕ measures angle with positive z-axis, $0 \le \phi \le \pi = 180^\circ$):

$$x = (\rho\sin\phi)\cos\theta, \quad y = (\rho\sin\phi)\sin\theta, \quad z = \rho\cos\phi, \quad dV = \rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$