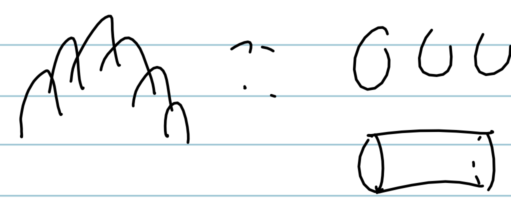
Math 200, Dec 4

Final!

- → No Lagrange w 2 constraints
- → No points for 3-D sketches (but you need to reason about 3-D regions somehow...)
- → No operation counting
- – etc. [to be posted]

Formula Sheet ! [to be posted]



Past exams do have

- 3-D sketching
- Lagrange 2 constraints

etc.

Find min/max of

$f = 2x^2 + 3y^2 - 4x - 5$

over disk $x^2 + y^2 \leq 16$

(Prob 2, 2004 WT2)

Find ∇f on interior, set to 0

Check boundary

$\nabla f = (f_x, f_y) = (4x-4, 6y)$

$\nabla f = (0,0) \Rightarrow 4x-4=0,\ 6y=0$

$x=1,\ y=0$

$\nabla f = \langle 0,0 \rangle$ in interior only at $(1,0)$

$f(1,0) = 2 + 0 - 4 - 5$

$= -7$

Boundary $(x^2 + y^2 = 16)$

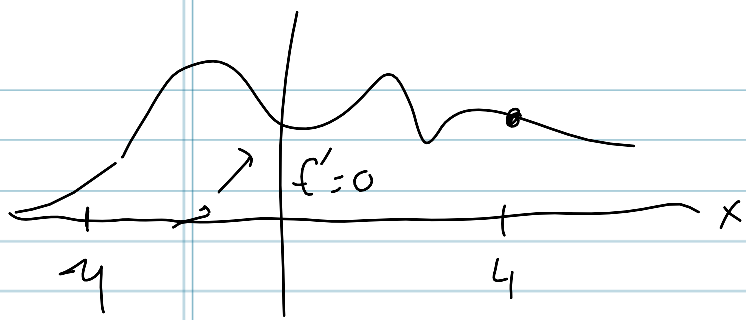
$x^2 = 16 - y^2$

$x = \pm\sqrt{16-y^2}$

$\tan^{-1} x = \tan^{-1}(\pm\sqrt{16-y^2})$

...

$f = 43 - x^2 - 4X$



Don't forget:

we get f at boundary of $x^2 + y^2 \le 16$ is

$43 - x^2 - 4x$, but $x \in [-4, 4]$

$43 - x^2 - 4x$ at

$x = -4$, $x = 4$, any interior point at which $(43 - x^2 - 4x)' = 0$

Lazy evaluation ☺

Direct "

$$f = 2x^2 + 3y^2 - 4x - 5$$

$$y^2 = 16 - x^2$$

on boundary $f =$

$$2x^2 + 3(16 - x^2) - 4x - 5$$

$$= 43 - x^2 - 4x$$

$$= (yz^2 - 2x,\ xz^2 + 2yz^3,\ xy2z + y^2 3z^2)$$

at $(-1, 1, 2)$

$$(4 - 2(-1),\ -4 + 2 \cdot 8,\ -4 + 12)$$

$$= (6, 12, 8)$$

$$\Delta f \approx \langle \nabla f \rangle \cdot \langle \Delta x, \Delta y, \Delta z \rangle$$

$$\langle 6, 12, 8 \rangle \cdot \langle \Delta x, \Delta y, \Delta z \rangle$$

2005 WT1

(4): Implicitly

$$xyz^2 + y^2 z^3 = 3 + x^2$$

find tangent plane at $(-1, 1, 2)$

$$f(x, y, z) = 0$$

$$f(x, y, z) = xyz^2 + y^2 z^3 - 3 - x^2 = 0$$

$$\nabla f = (f_x, f_y, f_z)$$

$z = z(x,y)$

$\Delta z = z_x \Delta x + z_y \Delta y$

$f_x \Delta x + f_y \Delta y + f_z \Delta z = 0$

if $f(x,y,z) = 0$

$\Delta z = \left(\dfrac{-f_x}{f_z}\right)\Delta x + \left(\dfrac{-f_y}{f_z}\right)\Delta y$

f = always 0

$\nabla f = 0$ so

$$\underbrace{6\,\Delta x}_{(x - x_0)} + \underbrace{12\,\Delta y}_{(y - y_0)} + \underbrace{8\,\Delta z}_{(z - z_0)} = 0$$

$(x - -1) \quad y - 1 \quad z - 2$

$6(x+1) + 12(y-1) + 8(z-2) = 0$

$z = \_\,x \quad \_\,y \quad \_$