Last time:

$dV = dz\,(r\,dr\,d\vartheta)$

integrand $= x^2+y^2 = r^2$

(polar)

$\sqrt{3}\,r \le z \le \sqrt{9-r^2}$

how is r bounded?

ϑ is anything in $[0, 2\pi]$

r is bounded by

$$\sqrt{3}\,r \le \sqrt{9-r^2}$$

$$3r^2 \le 9-r^2$$

$$4r^2 \le 9 \qquad 0 \le r \le 3/2$$

Math 200, Dec 2

2009 WT2

Problem 8:

$$I = \iiint_T (x^2+y^2)\,dV$$

bounded below by cone

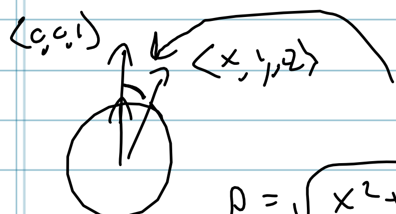
$$z = \sqrt{3x^2+3y^2}$$

i.e. $z \ge \sqrt{3x^2+3y^2}$

above by sphere

$$x^2+y^2+z^2 = 9$$

i.e. $z \le \sqrt{9-x^2-y^2}$



$$\rho = \sqrt{x^2+y^2+z^2}$$

$$= |\langle x,y,z\rangle|$$

Generally reasonable: $r = \rho\sin\varphi$

$x = r\cos\vartheta$

$y = r\sin\vartheta$

$x = (\rho\sin\varphi)\cos\vartheta$

$y = (\rho\sin\varphi)\sin\vartheta$

In cylindrical:

$$\int_{\vartheta=0}^{\vartheta=2\pi}\left(\int_{r=0}^{r=3/2}\left(\int_{z=\sqrt{3}r}^{z=\sqrt{9-r^2}} r^2\,dz\right) r\,dr\right) d\vartheta$$

reasonable..

$$\int\left(\sqrt{9-r^2}-\sqrt{3}\,r\right) r^3\,dr$$

$(\sqrt{9-r^2})(r^2)\cdot r \qquad r^4$

Spherical: $z = \rho\cos\varphi$, $r = \rho\sin\varphi$

$z \le \sqrt{9-r^2}$, i.e. below sphere

$\rho^2 \le 9$

$\boxed{0 \le \rho \le 3}$ 1

$\sqrt{3}\, r \le z$

$\sqrt{3}\, \rho \sin\varphi \le \rho \cos\varphi$

$\sqrt{3} \sin\varphi \le \cos\varphi$

$\tan\varphi = \dfrac{\sin\varphi}{\cos\varphi} \le \dfrac{1}{\sqrt{3}}$

φ	0	45°	$90^\circ = \frac{\pi}{2}$	
$\sin/\cos$	$0/1$	$\frac{1/\sqrt{2}}{1/\sqrt{2}}$	$1/0$	$\frac{1}{\sqrt{3}}$

Already!

Region T:

$0 \le \vartheta \le 2\pi$

$0 \le r \le 3/2$

$\underbrace{\sqrt{3}\, r \le z}_{\text{expect directly about } \varphi} \le \underbrace{\sqrt{9-r^2}}_{\text{sphere}}$ $\iff \rho = \text{const}$

$=$

$z \le \sqrt{9-r^2}$

$\underbrace{x^2+y^2+z^2}_{\rho^2}$ or $\underbrace{r^2+z^2}_{\rho^2} \le 9$

$$\int_{\vartheta=0}^{\vartheta=2\pi} \left(\int_{\varphi=0}^{\pi/6} \left(\int_{\rho=0}^{?} \left(\text{stuff inside}\right) d\rho \right) d\varphi \right) d\vartheta$$

$? = \min\left(3, \dfrac{3}{2\sin\varphi}\right)$

(since $\rho \le \dfrac{3}{2\sin\varphi}$ and $\rho \le 3$)

is $3 < \dfrac{3}{2\sin\varphi}$

or $3 > \dfrac{3}{2\sin\varphi}$

30° ! $\dfrac{\sin = 1/2}{\cos = \sqrt{3}/2} = \dfrac{1}{\sqrt{3}}$

$\theta =$	0°	30°	45	90°
$\tan = \sin/\cos$	0	$\frac{1}{\sqrt{3}}$	1	$+\infty$

$\boxed{0 \le \varphi \le 30^\circ}$ 2

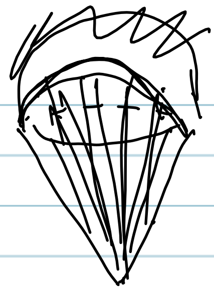
$r \le 3/2$

$\rho \sin\varphi \le 3/2$

$\boxed{\rho \le \dfrac{3}{2\sin\varphi}}$ 3

(usually $d\rho$ then $d\vartheta$ then $d\varphi$

spherical cap



ice cream cone

sketch it should be

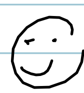
$0 \le \rho \le 3$

not

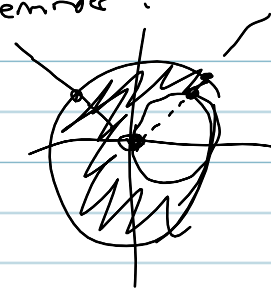
$0 \le \rho \le \frac{3}{2\sin\varphi}$

Try $3 \le \frac{3}{2\sin\varphi}$?

$(0 \le \varphi \le 30^\circ)$ $\sin\varphi \le \frac{3}{2 \cdot 3} = \frac{1}{2}$



Reminder:



$\vartheta \in [\frac{\pi}{2}, \frac{3\pi}{2}]$ one thing for r

$\vartheta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ something else

=

φ near 0: 3 vs $\frac{3}{2\sin\varphi}$

$\sin\varphi$ near 0

$3 \le \frac{3}{2\sin\varphi}$

$$\int_{\varphi=0}^{\pi/6}\int_{\vartheta=0}^{2\pi}\int_{\rho=0}^{3} \quad d\rho\, d\vartheta\, d\varphi$$

integrand $x^2+y^2 = r^2$ $\quad \rho^2 \sin\varphi$

$r = \rho\sin\varphi$

$r^2 = \rho^4 \sin^2\varphi$

$\rho^4 \sin^3\varphi$

$\sin\varphi \ \underbrace{\sin^2\varphi}_{1-\cos^2\varphi}$

so $0 \le \varphi \le 30^\circ$

sine ranges from $0 \to \frac{1}{2}$

$\sin\varphi$ always $\le \frac{1}{2}$

$3 \le \frac{3}{2\sin\varphi}$

$\rho \le 3$, and $\rho \le \frac{3}{2\sin\varphi}$

mean $\rho \le 3$

$$\int_{\varphi=A}^{\varphi=B}\int_{\vartheta=C}^{\vartheta=D}\int_{\rho=E}^{\rho=F}(\rho^4)(\sin^3\varphi)\,d\rho\,d\vartheta\,d\varphi$$

$$=\left(\int_{\varphi=A}^{\varphi=B}\sin^3\varphi\,d\varphi\right)$$ "independent"

$$\left(\int_{\vartheta=C}^{\vartheta=D}d\vartheta\right)\left(\int_{\rho=E}^{\rho=F}\rho^4\,d\rho\right)$$